COMPUTER ANALYSIS FOR DYNAMIC RESPONSE OF BEAMS AND PLATES ON ELASTIC FOU.. (U) PENNSYLVANIA STATE UNIV UNIVERSITY PARK DEPT OF CIVIL ENGINEE..

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COMPUTER ANALYSIS FOR DYNAMIC RESPONSE
OF BEAMS AND PLATES ON ELASTIC FOUNDATIONS

Prepared for

United States Air Force
Air Force Office of Scientific Research
Bolling AFB, DC 20332

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METHOD OF DIRECT ANALYSIS
PLATES ON ELASTIC FOUNDATION

DYNAMIC LOADING

BEAM ON ELASTIC FOUNDATION

The current Air Force nondestructive pavement Testing (NDPT) method requires a rational method to evaluate pavement layer modulus values. A reasonable step toward the development of such a method is computer analyses of elastic layered systems subjected to dynamic loading. This research adopts the Method of Direct Analysis which uses the impulsementum laws and constitutive relations but bypasses the explicit use of differential equations. As a result of the research, two computer programs were developed, one for infinite beams and the other for infinite

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ABSTRACT

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NOMENCLATURE

- A_i = Cross-sectional area which contributes to dynamic inertia.
- A_{q} = Cross-sectional area which contributes resistance to shearing.
- $C_p = Plate velocity = {E/[v(1-v^2)]}^{\frac{1}{2}}$
- $C_1 = Dilatational$ wave velocity in a beam = $[EI_b/\rho I_i]^{\frac{1}{2}}$
- C_2 = Shear wave velocity in a beam = $[A_sG/\rho A_1]^{\frac{1}{2}}$
- $C_2^1 = \text{Shear wave velocity in a plate} = [G/\rho]^{\frac{1}{2}}$
- D = Flexural rigidity of plate = $Eh^3/[12(1-v^2)]$
- E = Modulus of elasticity
- G = Modulus of rigidity = E/[2(1+v)]
- h = Plate thickness
- I_{h} = Moment of inertia which contributes resistance to bending
- \mathbf{I}_{i} = Moment of inertia which contributes resistance to dynamic inertia
- j = Superscript referring to quantities of the jth cell
- k, = Shear correction factor for plate
- M = Internal bending moment
- M_r = Radial bending moment per unit length
- $\mathbf{M}_{\mathbf{A}}$ = Tangential bending moment per unit length
- q = Intensity of distributed external load on beam
- r = Radial distance along plate

- t = Time
- V = Vertical shear force on a cross-section of the beam
- $v = Velocity of deflection in a beam or plate, <math>y_t$ or w_t , respectively
- w = Transverse displacement of the midplane of plate
- x = Coordinate along length of beam
- y = Deflection of beam
- $\varepsilon_{_{\mathbf{U}}}$ = Total slope of the deflection curve of the beam
- $\varepsilon_{,h}$ = Angular strain of an element of the beam
- $\epsilon_{\rm p}$ = Total slope of the deflection curve of the plate
- ε_{\star} = Angular strain of an element of the plate
- θ = Tangential direction
- ν = Poisson's ratio
- ρ = Density of the material of beam or plate
- φ = Rotation of the cross-section of the plate about the tangential axis
- ψ = Slope of the deflection curve of a beam when shearing force is neglected
- ω = Angular velocity of rotation of an element of the beam or plate, $\psi_{\rm t} \; {\rm or} \; \phi_{\rm t}, \; {\rm respectively}$

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1. INTRODUCTION

1.1 Background Information

The current Air Force Nondestructive Pavement Testing (NDPT) method contains two main components—data collection equipment and analytical method [1]. The data collection equipment is used primarily for evaluation of in—situ elastic modulus of each pavement constituent material. The elastic modulus values are then used as input data for the analytical method which determines the structural capacity of existing airfield pavements.

The data collection equipment consists of an impulse loader with the necessary instrumentation and a desk-top computer for preliminary data analysis and evaluation. The pavement response to the impulse loading is measured by using accelerometers; the acceleration-time data are analyzed by using the Discrete Fourier Transform (DFT) technique [2] to obtain the phase angle versus frequency relationship which is then used to develop the dispersion curve. From the dispersion curve, the shear wave velocity propagating through the pavement is obtained and the shear modulus of each pavement constituent material is computed. Although the computation of elastic modulus is simple, the interpretation of dispersion curves for modulus calculation is not easy and straightforward [3,4]. For this reason, the current NDPT method is without an adequate method for computation of layer modulus values [5].

Pavement response to dynamic loading is a complex problem due to the presence of layers of different materials and also the nature of the constituent materials. The presence of layers causes reflection and diffraction of waves. Under dynamic loading, the pavement materials such as bituminous concrete and soil may produce real and imaginary modulus components which vary with frequency of vibration [6]. Because of these factors, it is very difficult to find a solution especially for high order modes of response [7]. As a consequence, no straightforward procedure is presently available for determination of realistic modulus values from dynamic loading test results [1].

However, there is the so-called method of Direct Analysis which bypasses the explicit use of differential equations and may avoid some of the difficulties just mentioned. Because this method has been successfully applied to certain beam and plate problems [8,9], this research is undertaken to investigate the feasibility of using this technique to analyze the response of elastic layered system to dynamic loading. While the ultimate goal of the research is to develop a computer program for elastic layered system, the immediate objective of this study is specified below.

1.2 Research Objective

The ultimate goal of the research was to develop a computer program for analysis of the response of elastic layered system to dynamic loading. As a first step striving toward the goal, this

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research was undertaken to test the available theory and to develop the modifications necessary for direct application of the Method of Direct Analysis to problems of infinite beams and plates on elastic foundation.

2. INFINITE BEAM ON ELASTIC FOUNDATION

2.1 The Physical Laws

The physical laws used in the development of computer programs include the impulse-momentum laws and the constitutive relations. The beam is divided lengthwise into elements of equal length. For each element, the equations of motion and constitutive relations are derived for two modes of motion — rotation and translation. The basic assumptions required in the derivation of the physical laws include that the material is elastic, homogeneous and isotropic, that the deformation of the beam is due to both flexural motion and shear deflection of the cross-section, and that the elastic foundation behaves as Winkler foundation. These basic assumptions may be found elsewhere [10].

2.1.1 Equations of Motion

The equations of motion are obtained by applying the impulse-momentum laws directly to an element of the beam. Figure 1 shows the free-body diagram of a typical jth element. For this element, the rates of change in both rotation and translation are given below:

Rotation of the jth element -

$$\frac{d\omega}{dt} = \frac{1}{\alpha I_{i} dx} \left[1/2 \left(\nabla^{j} + V^{j+1} \right) + (M^{j} - M^{j+1}) \right]$$
 (1)

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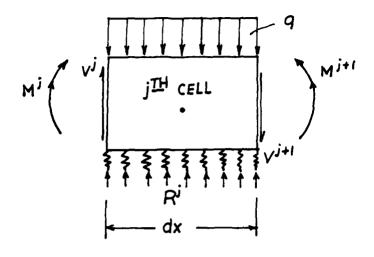


Figure 1. Free-Body Diagram of jth Element

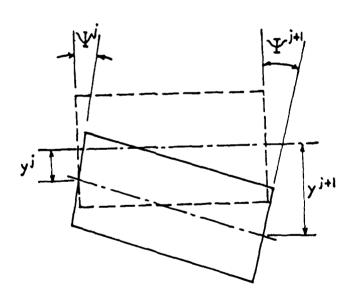


Figure 2. Deformation of jth Element

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Translation of the jth element -

$$\frac{d\mathbf{v}}{d\mathbf{t}} = \frac{1}{\rho \mathbf{A_i} d\mathbf{x}} (\mathbf{v^{j+1}} - \mathbf{v^j} + \mathbf{q} d\mathbf{x} - \mathbf{R_j})$$
 (2)

The foundation reaction, R_j, shown in Eq. (2) is a function of foundation deformation which, in essence, is the deflection of the beam.

Two different foundation supports are considered in the computer program, i.e., linear and nonlinear supports. For the linear support, the foundation reaction is directly proportional to the deflection, viz

$$R_{i} = -ky_{j} \tag{3}$$

For the nonlinear support, the following hyperbolic function is used:

$$R_{i} = -ky_{i}/(1 + ny_{i})$$
 (4)

where k is the spring constant having a unit of psi, y_j is the mean deflection of the jth element and n is a constant with a unit of in⁻¹. The above hyperbolic relationship between load and deformation is often used for describing the nonlinear behavior of soils [11,12].

2.1.2 Constitutive Equations

The constitutive equations are obtained from the consideration of the deformation of a typical beam element. As shown in Figure 2, the jth element undergoes flexural as well as shear deformation. Thus

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two equations can be obtained, one for shear force and the other for the bending moment. According to Timoshenko beam theory [10], the shear force is proportional to the shear angle, namely

$$V = As G \{ \epsilon_{\mathbf{y}} - \psi \}$$
 (5)

where

$$\varepsilon_{y} = (y^{j+1} - y^{j})/dx \tag{6}$$

For the internal bending moment, the simpler Euler-Bernoulli theory provides the following relationship:

$$M \approx -EI_b \{ \epsilon_{\psi} \} \tag{7}$$

where

$$\varepsilon_{\psi} = (\psi^{j+1} - \psi^{j})/dx \tag{8}$$

It has been shown [13] that any input to the beam which causes it to undergo a flexural mode of motion immediately produces two different waves — dilation and shear waves. The dilation wave which is caused by discontinuities in ω and M, as well as discontinuities in their higher derivation propagates at a velocity C_1 which is equal to

$$C_1 = \left[\frac{EI_b}{\rho I_4}\right]^{\frac{1}{2}} \tag{9}$$

~._{*}

The shear wave is related with discontinuities in v and V, as well as discontinuities in their higher derivatives. The shear wave velocity may be shown to be [13]

$$C_2 = \left[\frac{GA_s}{\rho A_t}\right]^{\frac{1}{2}} \tag{10}$$

2.1.3 Boundary Conditions

The problem under investigation is a beam on elastic foundation which has infinite length and is subjected to a vertical dynamic load. The dynamic load is distributed over a small area which can be treated as a concentrated load. In the analysis, the beam is split into two at the loading point so that each semi-infinity beam carries a shear force which is equal to one-half of the applied load. Meanwhile, for the continuity requirement, the slope of the deformed beam at the loading point is maintained at a value of zero.

Furthermore, the shape of the deflected beam dictates that deflections beyond distance L (measured from the load) become very small. According to Timoshenko [14], the distance L for static loading is approximately equal to $5.5/\beta$, where $\beta=4\sqrt{k/4\rm EI}$, k = spring constant of the elastic foundation, E = Young's modulus, and I = moment of inertia of the cross section of the beam with respect to z axis. On this basis, the semi-infinite beam can be approximated by a beam with a finite length L. Thus, for the problem under consideration, the shear input and boundary conditions are

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$$V = -\frac{P}{2}$$
 (11)

$$\psi \bigg]_{\mathbf{x}=\mathbf{0}} = 0 \tag{12}$$

$$y \bigg]_{\kappa=L} = 0 \tag{13}$$

In the analysis, it is convenient to express the boundary conditions in terms of the angular and linear velocities, respectively, as follows.

$$\omega \bigg]_{\mathbf{x}=\mathbf{0}} = \psi_{\mathbf{t}} \bigg]_{\mathbf{x}=\mathbf{0}} = 0 \tag{14}$$

$$v\Big]_{x=L} = y_t\Big]_{x=L} = 0$$
 (15)

where subscript t denotes time.

The preceding equations are a complete statement of the problems of the flexural traveling stress waves in an infinite beam on elastic foundations. Of these equations, the rotational impulse-momentum law, Eq. (1), indicates that the angular velocity of rotation, ω , is a function of both the moment and the shear. Since the moment is associated with the dilation wave and the shear with the shear wave, Eq. (1) exhibits an interaction of two waves and by definition the angular velocity is a coupled quantity. Moreover, Eq. (5) indicates that the shear force is also a coupled quantity since it is a function of both the linear displacement gradient

and the angle of rotation. Because of the interaction or coupling between the two waves, variables which are associated with both wave speeds must be given special treatment. Thus, the solution requires a special technique of coupling which is presented in the following.

2.2 The Technique of Coupling

In the Method of Direct Analysis, the beam is divided into finite number of elements or cells as mentioned earlier. It is convenient to determine the cell length based on the "characteristic assumption," namely, dx = c dt [8,15], where c = wave velocity and dt = time increment. Since two distinct wave speeds are present, the cell length dx = c dt cannot be satisfied for both speeds simultaneously. Thus, a technique is needed to determine whether the wave under consideration has passed across the cell. If the wave has crossed the cell, the computation of stresses, strains and velocities for this cell will be triggered. Otherwise, further computation is skipped and the current dynamic variables are held over until the time has sufficiently advanced, and the cycle is repeated.

There are two procedures available for carrying out the above technique, namely, the "wave index" and "clock" methods [16]. These two procedures are completely analogous but the "clock" method is more amenable to a problem in which two waves are present. For this reason, the "clock" method is adopted in this analysis and is discussed below.

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In the "clock" method, two time "clocks" are established; one for dilatation wave and the other for shear wave. Each "clock" signals the instant when the wave propagation procedure for that particular wave is undergone. The rate of advancement of the two clocks are defined as follows:

$$dt_{D} = \frac{dx}{C_{D}} \tag{16}$$

for dilatation wave clock, and

$$dt_{R} = \frac{dx}{C_{S}} \tag{17}$$

for shear wave clock,

in which $\mathbf{C}_{\mathbf{D}}$ and $\mathbf{C}_{\mathbf{S}}$ are the dilatation and shear wave velocities, respectively.

Since C_D is greater than C_S , the dilatation wave front initially will reach the end of a cell before the corresponding shear wave front. When this has happened, the dilatation time clock will advance dt_D , viz, $\mathrm{t}_D' = \mathrm{t}_D + \mathrm{dt}_D$, and the quantities associated with the dilatation wave will be adjusted. Then, two possibilities follow. One possibility is that the dilatation clock has advanced another dt_D before the shear clock. When this condition occurs, the quantities associated with the dilation wave front again will be adjusted. The other possibility is that the shear wave clock advances dt_R before the next advancement of the dilatation clock. When this happens, the quantities associated with

the shear wave front will be adjusted. A flow diagram summarizing this propagation procedure is given in Figure 3. Note that in this analysis, the actual time, t, is chosen to coincide with t_D . A more detailed discussion on wave propagation procedures is available elsewhere [16].

The preceding wave propagation technique is used as the basis for the arrangement of the various physical laws presented earlier. It should be emphasized that the position of the important steps must be carefully organized in order to maintain a stable solution. In general, the arrangement requires that the cause and effect are properly sequenced and the statement of boundary conditions should follow immediately after the calculation of the dynamic quantity for which they are prescribed.

2.3 The Technique of Considering Damping

The effect of damping is taken into consideration with the use of exponential damping function. In the problem of flexural traveling waves in beams and plates there are two velocities — linear and angular velocities which must be damped in order to attain an equilibrium state. According to exponential damping function which is often used, these two velocities can be expressed as follows:

$$\omega = \omega_0 e^{-t/\tau_1} \tag{18a}$$

$$\mathbf{v} = \mathbf{v}_0 e^{-\mathbf{t}/\tau_2} \tag{18b}$$

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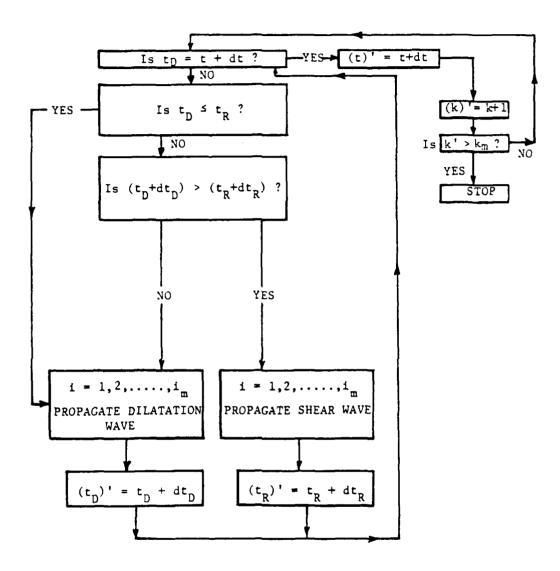


Figure 3. Flow Diagram for "Clock" Method of Wave Propagation

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where ω_0 and v_0 are the initial linear and angular velocities, respectively, and τ_1 and τ_2 are arbitrary constants.

Upon differentiation, one obtains the magnitude of damping occurring in time dt or the change of angular and linear velocities due to damping as follows:

$$d\omega = -\omega \frac{dt}{\tau_1} \tag{19a}$$

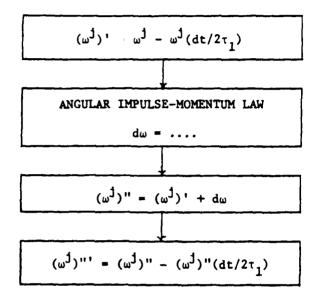
$$dv = -v \frac{dt}{\tau_2}$$
 (19b)

These damping relations Eq. (19a and b) are incorporated into the previously developed impulse-momentum laws as shown in Figure 4. It should be noted that because of the desire to eliminate computation fluctuations, it has been found necessary to use half of the initial and half of the final velocity (linear or angular) to obtain the total contribution of motion damping occurring in time dt.

From Eq. (19) it is seen that a completely undamped solution can be obtained by letting τ_1 and τ_2 equal to infinity. To obtain a transient and static solutions, $\mathrm{dt/\tau_i}$ must be restricted to lie in the range between zero and one. This general principle of synthesizing static and dynamic solutions constitutes a unique feature of the Method of Direct Analysis. Note that in mathematical approaches, static and dynamic solutions are governed by elliptic and hyperbolic equations, respectively. Separate analyses are generally required for each type of problem.

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Dilatation Wave (angular velocity)



Shear Wave (linear velocity)

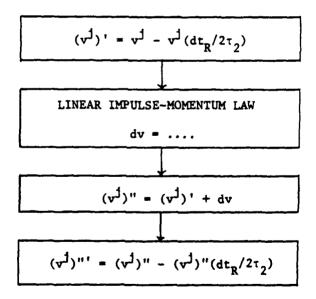


Figure 4. Flow Piagram for Damping Consideration

Using these properly organized physical laws, boundary conditions and damping equations, the computer program developed by Koenig [16] for cantilever beams has been modified and extended to suit the condition of infinite beams on elastic foundations. The computer program which is developed for IBM 370/3081 is capable of providing the response of beams to both transient and steady-state loadings. The transient loading may be a triangular, rectangular or haver sine pulse; and the steady-state loading may include harmonic or sustained loadings. Furthermore, both linear and non-linear elastic foundation can be considered.

The computed response can be presented in both numerical and graphical forms. The plotting of output is accomplished by incorporating an available program PLOTIT [17] into the computer program. The listing of the complete computer program is included in Appendix A.

3. DYNAMIC RESPONSE OF BEAM ON ELASTIC FOUNDATION

The developed computer program was used to analyze the response of infinite beam on elastic foundation for several types of dynamic loading to demonstrate its effectiveness in the analysis of stress wave propagation. The dynamic loadings analyzed included step sustained loading, step pulse loading and sinusoidal loading. Results of the analyses are presented and discussed below.

3.1 Response to Step Sustained Loading

In this analysis, the infinite beam is made of portland cement concrete having a cross section 1 in. wide and 4 in. high with a modulus of elasticity of 3 x 10⁶ psi. The beam is subjected to a concentrated vertical load of 200 pounds with a sufficiently long duration so that it can be considered as a sustained load. The beam is supported by springs having two different stiffnesses, one lower and the other higher than the stiffness of the beam. To demonstrate the ability of the computer program for handling different damping conditions, both under and over-damped conditions are analyzed for one spring stiffness.

3.1.1 Over-Damped System

In the preceding chapter, it has been pointed out that for over-damped condition, the constant τ_i in Equation (19) must be small. It should be noted, however, that too small values of τ_i may over-

depress the response and as a result considerably longer time is needed to reach a steady-state condition. The time required to reach a steady-state response depends greatly on the stiffness of the beam-foundation system and time increment (dt) used in the analysis. Thus, the values of τ_i must be carefully chosen in order to keep the time to steady-state, and therefore the computation cost, to a minimum.

In the analysis, two spring stiffnesses, 1×10^4 and 4×10^6 psi, are used. For the system with 1×10^4 psi spring constant, the length of the beam analyzed is 48 in., cell length is 1 in. and the time constant τ_1 is chosen at 30 µsec. Results of the analysis are presented in two general forms: response versus time and response versus distance. Figures 5, 6 and 7 show the deflections, moments and shears at three locations versus time, respectively. It is seen that the responses almost reach a maximum at the end of the curves. Also, time lags exist between the responses at the loading point and that away from it indicating the phenomenon of stress wave propagation away from the loading point.

The computed responses shown at the end of the curves are plotted against distance in Figures 8 (deflection), 9 (slope), 10 (moment) and 11 (shear). Also included in these figures are an available exact solution for static loading which is given by Timoshenko and is described below [14].

Deflection
$$y = \frac{P\beta}{2k} e^{-\beta x} (\cos \beta x + \sin \beta x)$$
 (20a)

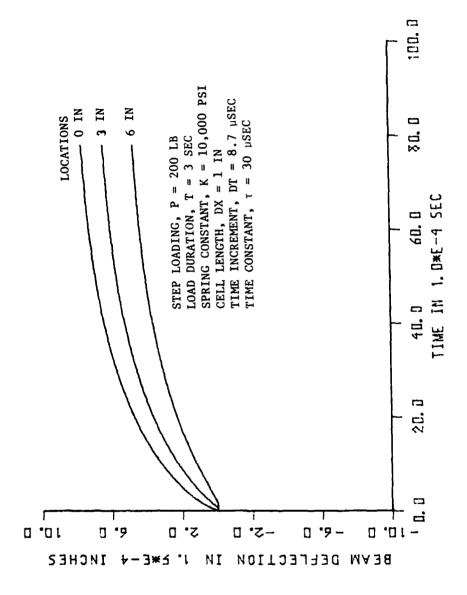


Figure 5. Variation of Beam Deflections with Time for Three Locations Measured from Loading Point, Spring Constant = 10^4 psi.

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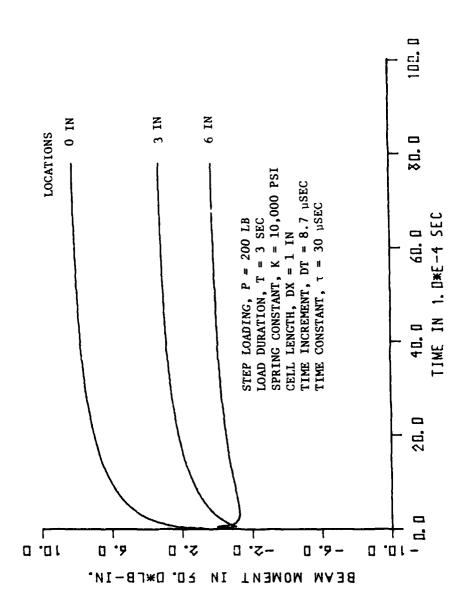


Figure 6. Variation of Beam Moment with Time for Three Locations Measured from Loading Point, Spring Constant = $10^4~\rm psi$.

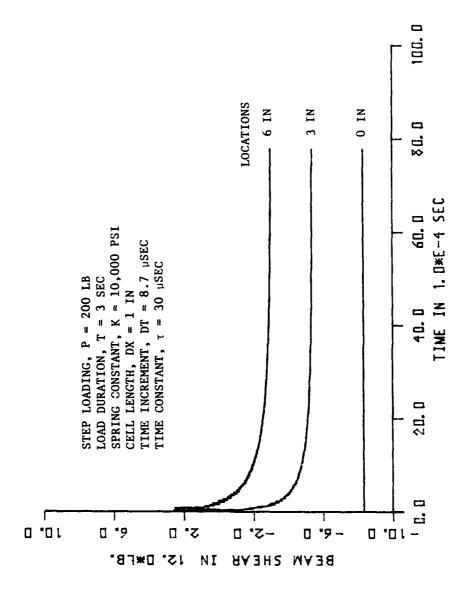
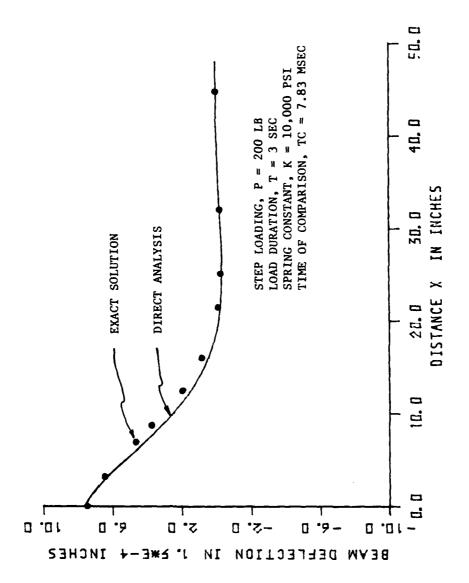


Figure 7. Variation of Beam Shear with Time for Three Locations Measured from Loading Point, Spring Constant = 10^4 psi.



Variation of Deflection with Distance and Comparison with Exact Solution. Figure 8.

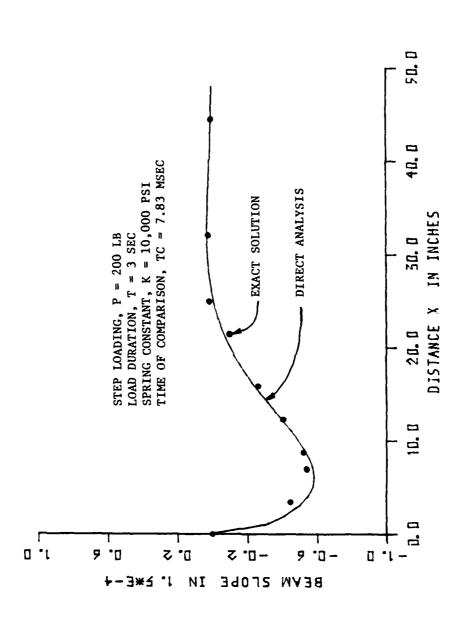


Figure 9. Variation of Slope with Distance and Comparison with Exact Solution.

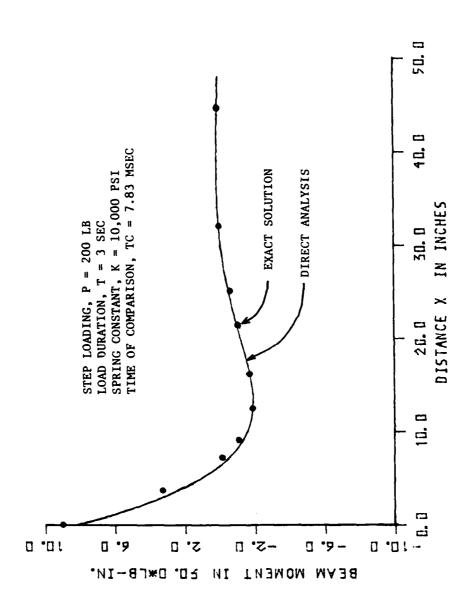


Figure 10. Variation of Moment with Distance and Comparison with Exact Solution.

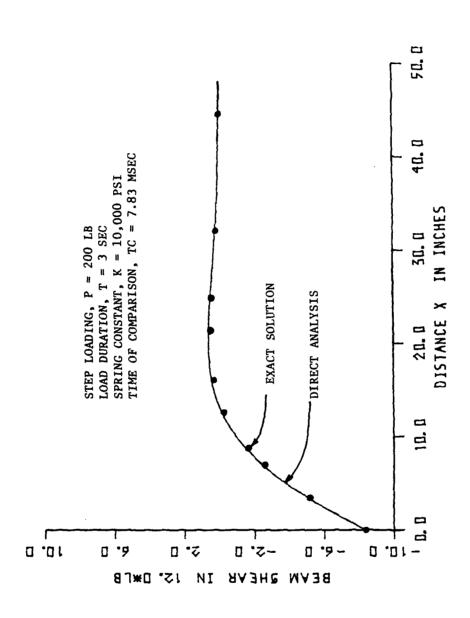


Figure 11. Variation of Shear with Distance and Comparison with Exact Solution.

Slope
$$\frac{dy}{dx} = -\frac{P\beta^2}{k} e^{-\beta x} \sin \beta x$$
 (20b)

Moment
$$M = -\frac{P}{4\beta} e^{-\beta x} (\sin \beta x - \cos \beta x)$$
 (20c)

Shear
$$V = -\frac{P}{2} e^{-\beta x} \cos \beta x$$
 (20d)

where notations P, k and β have been defined in the preceding chapter.

It is obvious that the exact solution provides continuous curves. For clarity, however, only some values are selected arbitrarily for comparison. Although there is some discrepancy between the two sets of data, the agreement generally speaking is very good. Primary causes for the difference may include the following: (1) At the time of comparison, the steady-state response has not yet established as revealed by Figures 5, 6 and 7 in which the response has not quite reached the constant value, (2) The cell length of 1 in. is too large. Of the four figures under consideration, Figure 9 provides a better view of the effect of coarse cell; in this figure, 1-in. line segments are clearly shown. It is believed that with the use of smaller cell length together with larger time of comparison, the results of Direct Analysis should match with the exact solution very well. The excellent agreement between the two sets of results indicates that the developed computer program is capable of providing accurate solution for infinite beam on elastic foundations.

Another system analyzed has a spring constant of 4×10^6 psi. For this condition, the length of the beam used is 10 in., cell length

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is 0.05 in., and time constant τ_i is chosen at 6.6 usec. Results of the analysis are summarized in Figures 12, 13 and 14 which show the variation of deflection, moment and shear with time, respectively. As expected, the shape of the curves resembles that of the previous system. Primary reasons for analyzing a system with such a high spring constant are two-fold: (1) to demonstrate that the computer program can be used to analyze the dynamic response of an electric beam supported by a foundation which is stiffer than the beam, and (2) to provide response data of over-damped condition for comparison with the data of under-damped condition which is presented later.

3.1.2 Under-Damped System

In order to obtain at least one full cycle of fluctuation and at the same time to keep the computation cost to a minimum, a very high value of spring constant (4 x 10⁶ psi) was selected for analysis.

This value of spring constant was also used in the preceding over-damped condition so that a comparison of response data between over-damped and under-damped systems can be made. As before, the length of the beam used in the analysis is 10 in., and the cell length is 0.05 in. but time constant is selected at 30 msec. Figures 15, 16 and 17 present respectively, the variations of deflection, moment and shear with time for three different locations. Although only a little more than one cycle of data are is obtained, a trend that the response data fluctuate around the maximum value of the over-damped solution (Figures 12 through

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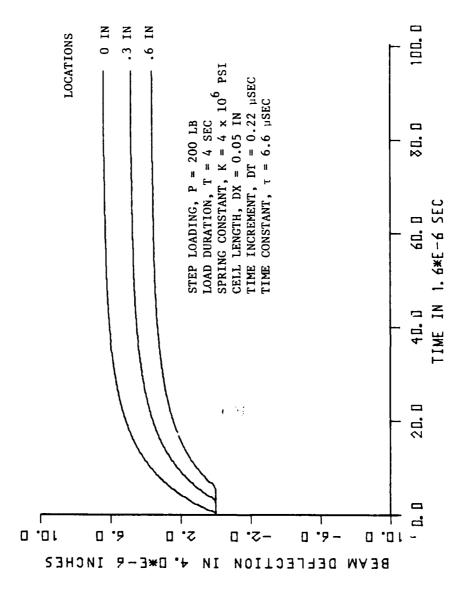


Figure 12. Variation of Beam Deflection with Time for Three Locations Measured from Loading Point, Spring Constant = 4 x 10^6 psi, Over Damped.

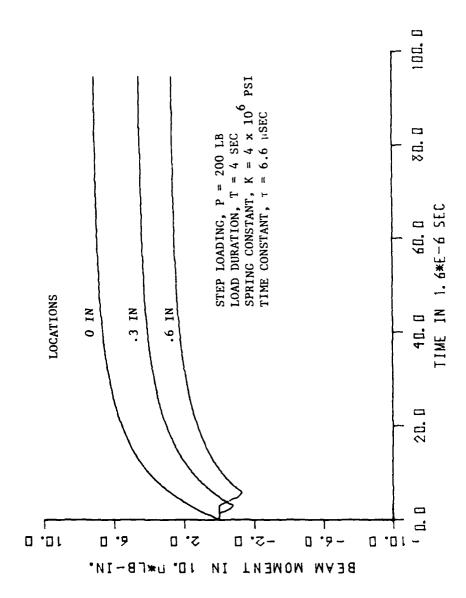
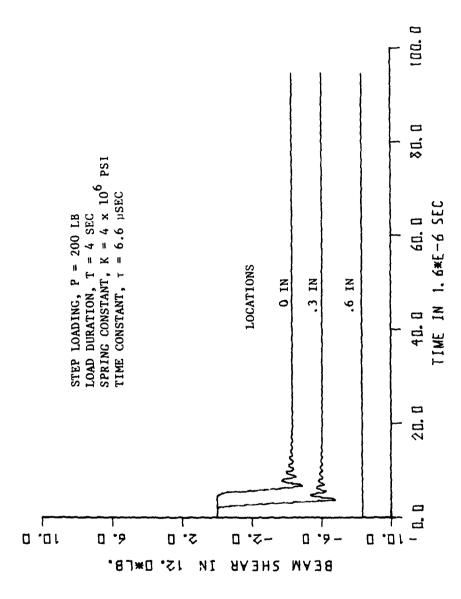
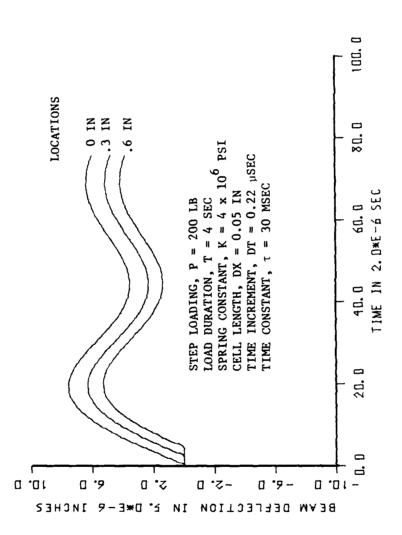


Figure 13. Variation of Beam Moment with Time for Three Locations Measured from Loading Point, Spring Constant = $4 \times 10^6 \, \mathrm{psi}$, Over Damped.

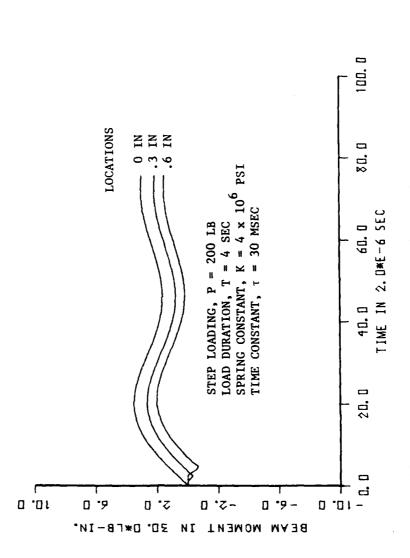


Variation of Beam Shear with Time for Three Locations Measured from Loading Point, Spring Constant $\approx 4 \times 10^9$ psi, Over Damped. Figure 14.



Variation of Beam Deflection with Time for Three Locations Measured from Loading Point, Spring Constant = 4×10^6 psi, Under Damped. Figure 15.

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Variation of Beam Moment with Time for Three Locations Measured from Loading Point, Spring Constant = $4 \times 10^6 \, \mathrm{psi}$, Under Damped. Figure 16.

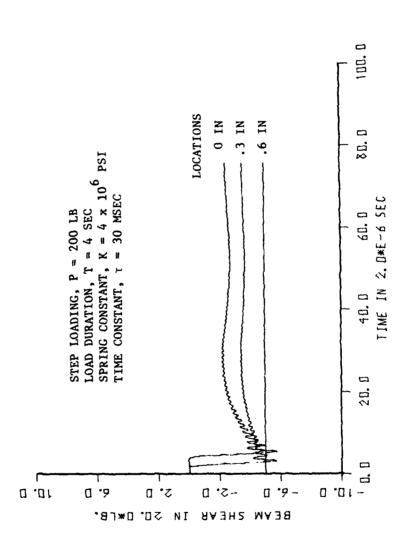


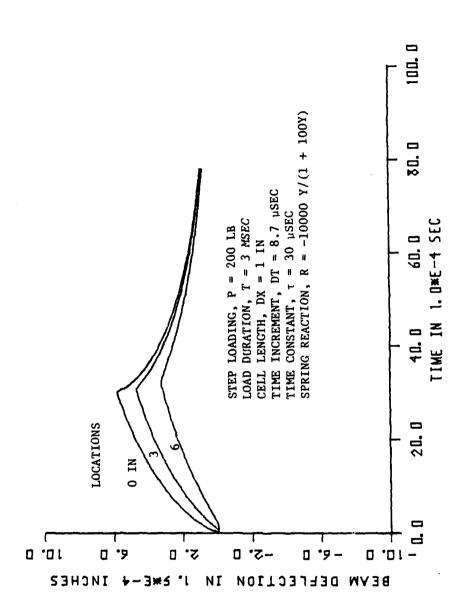
Figure 17. Variation of Beam Shear with Time for Three Locations. Measured from Loading Point, Spring Constant = 4×10^6 psi, Under Damped.

14) is revealed. Also, because of the damping, the peak response value decreases with increasing cycle of fluctuation.

3.2 Response to Step Pulse Loading

The step pulse loading analyzed is a 200 lb. vertical concentrated load having a load duration of 3 msec. The infinite beam is of the same material and the same dimension as that used in the previous condition, namely, portland cement concrete beam, 1 in. wide by 4 in. high with 3×10^6 psi modulus. However, the support is a nonlinear elastic foundation which behaves according to the hyperbolic function described by Eq. (4). In Eq. (4), the values of constants k and n are chosen to be 10,000 psi and 100 in. $^{-1}$, respectively. With the use of 1 x 10^4 psi for k value and a load of 200 lb., this system is different from the previous one only in load duration and nonlinear spring support. To obtain an over-damped solution, the same time constant $\tau = 30$ µsec as used before is adopted. Meanwhile, the length of the beam analyzed is also 48 in. and the cell length is 1 in.

The deflection, moment and shear versus time relationships obtained from the computer analysis are presented in Figures 18, 19 and 20, respectively. It is seen that during the loading period the response increases with time following the same path as that of the step sustained loading shown in Figures 5, 6 and 7. After the load is removed, i.e., after 3 msec, the response gradually diminishes with increasing time. A comparison between Figures 4 through 7 and Figures 18 through 20



Variation of Beam Deflection with Time for Three Locations Measured from Loading Point, Nonlinear Spring Support. Figure 18.

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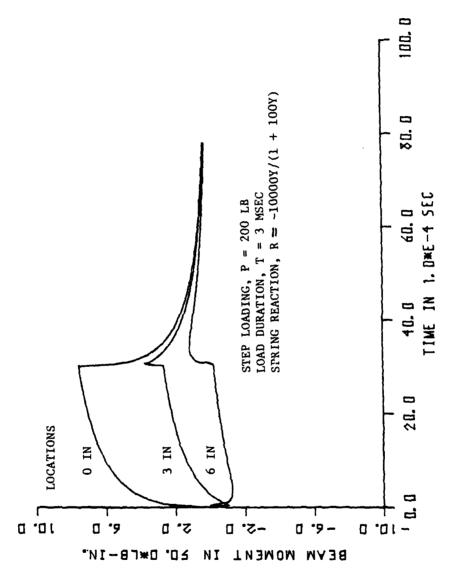
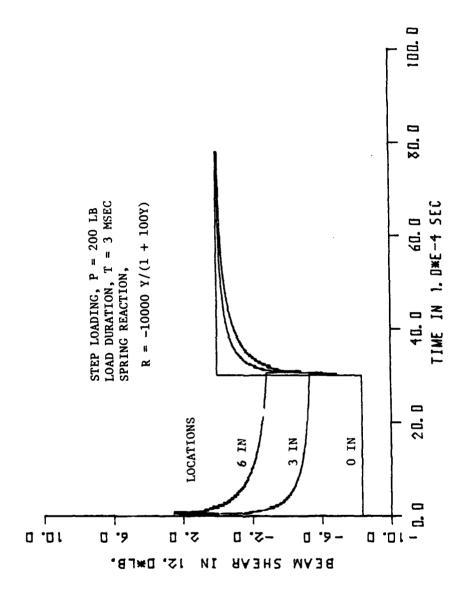


Figure 19. Variation of Moment with Time for Three Locations Measured from Loading Point, Nonlinear Spring Support.



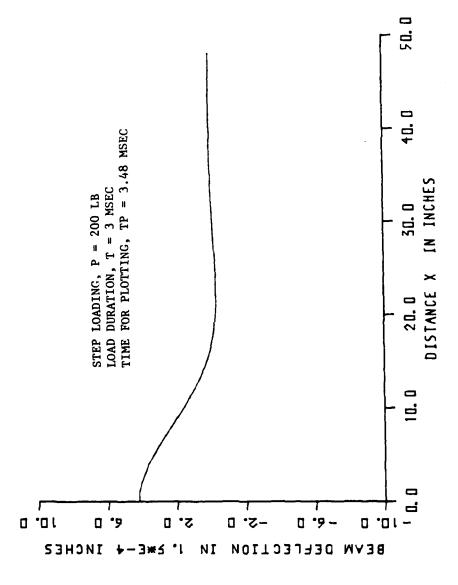
Variation of Shear with Time for Three Locations Measured from Loading Point, Nonlinear Spring Support. Figure 20.

indicates no apparent difference in the response within the loading period between the linear and nonlinear systems. This is probably because the magnitude of deflection under 200 lb. is not large enough to cause significant difference in foundation reaction.

To show the variation of response with distance, a time of 3.48 msec is used for plotting. The curves thus obtained are presented in Figures 31 (deflection), 22 (slope), 23 (moment) and 24 (shear). The shapes of these curves in general are similar to those in Figures 8 through 11 except for the shear curve especially in the region near the loading point. At x=0, the shear equals 100 lb. in Figure 11 and 0 lb. in Figure 24. This is due to the fact that the time used for plotting (3.48 msec) exceeds the duration of the pulse loading (3 msec). At 3.48 msec, the loading has already vanished and therefore the shear at x=0 equals zero as indicated in Figure 20.

3.3 Response to Sinusoidal Loading

The same portland cement concrete beam, 1 in. wide by 4 in. high, supported by a linear elastic foundation with $k = 1 \times 10^4$ psi is subjected to a sinusoidal loading which has an intensity of 200 lb. with a frequency of excitation of 3600 t. The duration of the sinusoidal loading is 3 sec. which is large enough to provide a steady-state response. As before, the length of the beam analyzed is 48 in. and the time constant is selected at 30 usec to effect over-damping. However, to reduce the cost



Variation of Beam Deflection with Distance, Nonlinear Spring Support. Figure 21.

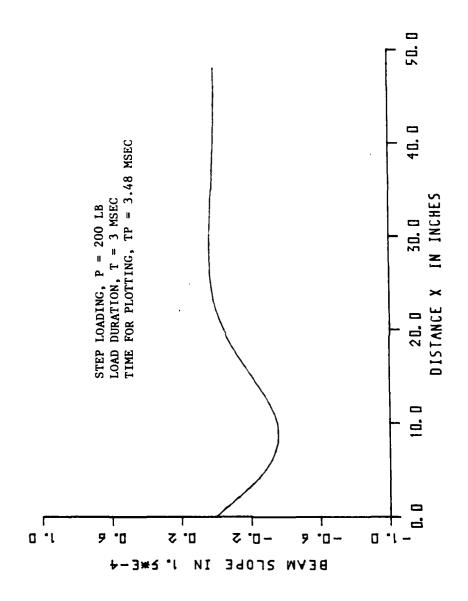


Figure 22. Variation of Slope with Distance, Nonlinear Spring Support.

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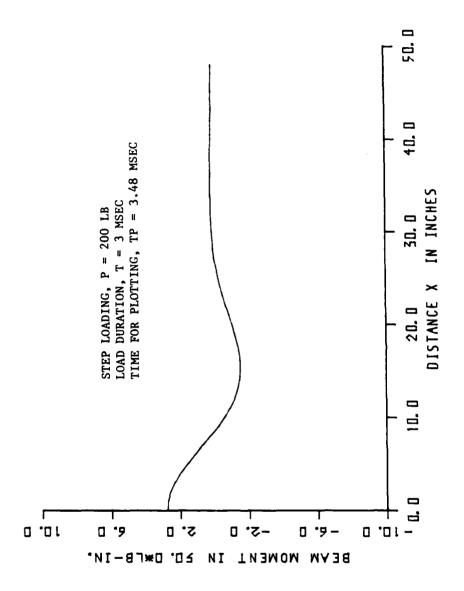


Figure 23. Variation of Moment with Distance, Nonlinear Spring Support.

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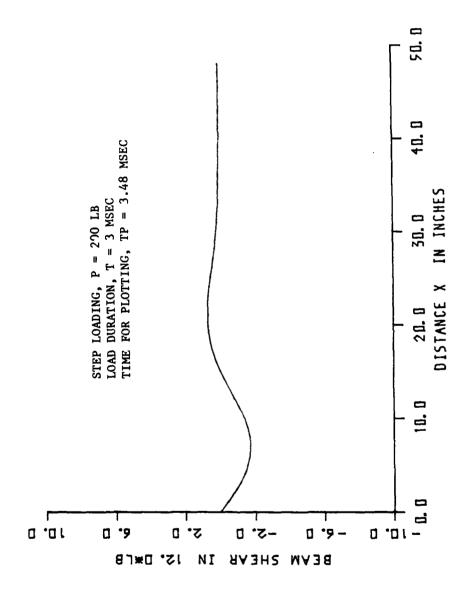
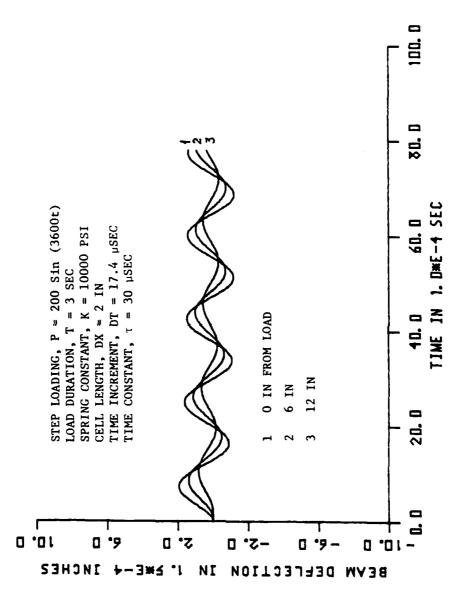


Figure 24. Variation of Shear with Distance, Nonlinear Spring Support.

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of computation a cell length of 2 in. instead of 1 in. is used in this analysis. The computed deflection, moment and shear are shown respectively in Figures 25, 26 and 27 as a function of time. As would be expected, delayed responses at points away from the load are shown.

The responses obtained for three different times, 8.7, 17.4 and 26.1 usec are plotted against distance in Figures 28 through 31. The time used for plotting is chosen arbitrarily. These figures demonstrate again the phenomenon of wave propagation.



Variation of Beam Deflection with Time for Three Locations Measured from Loading Point. Figure 25.

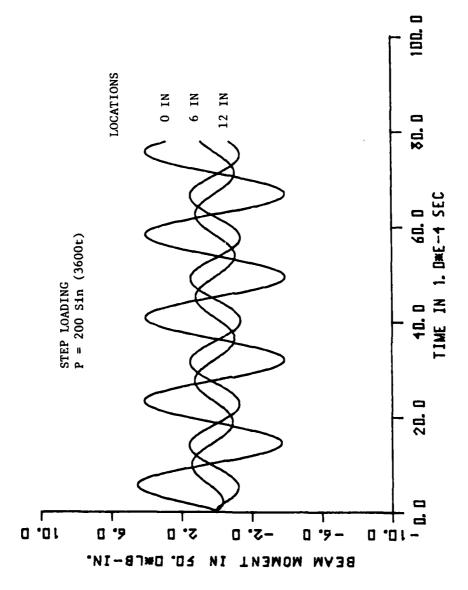


Figure 26. Variation of Beam Moment with Time for Three Locations Measured from Loading Point.

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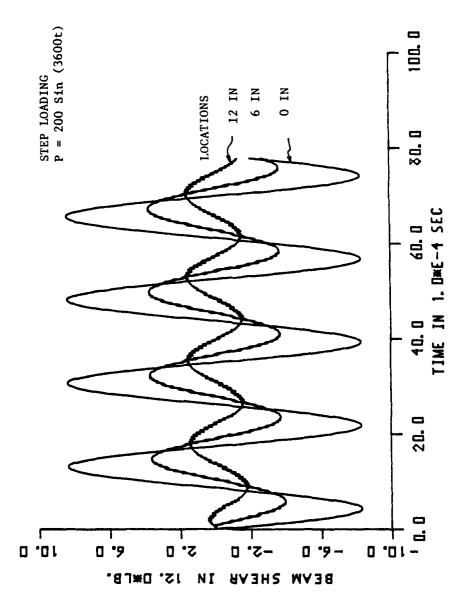
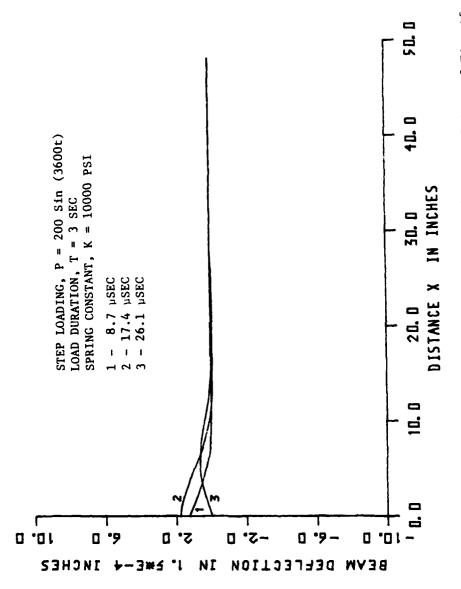


Figure 27. Variation of Beam Shear with Time for Three Locations Measured from Loading Point.

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Variation of Deflection with Distance for Three Points of Time After Load Application. Figure 28.

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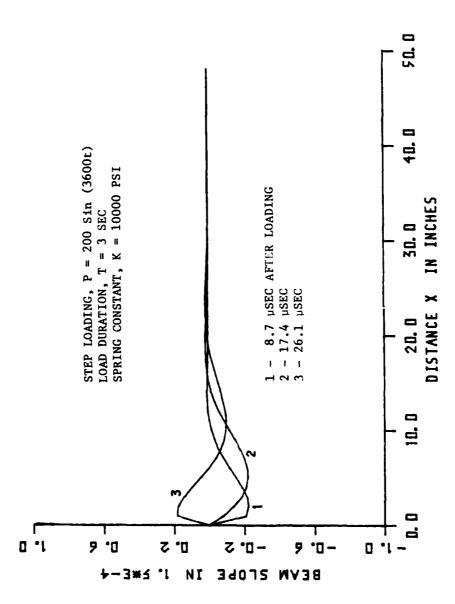


Figure 29. Variation of Slope with Distance for Three Points of Time After Load Application.

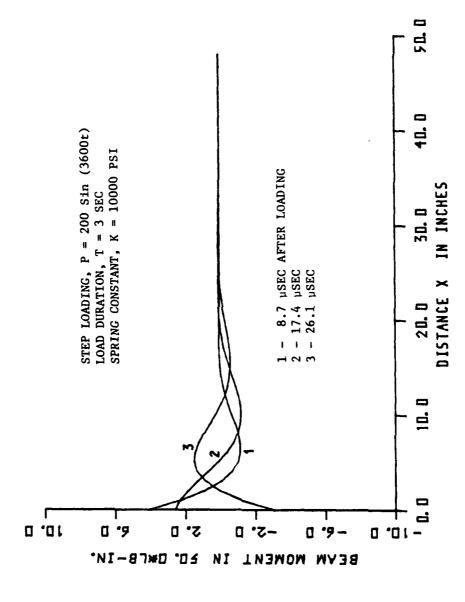
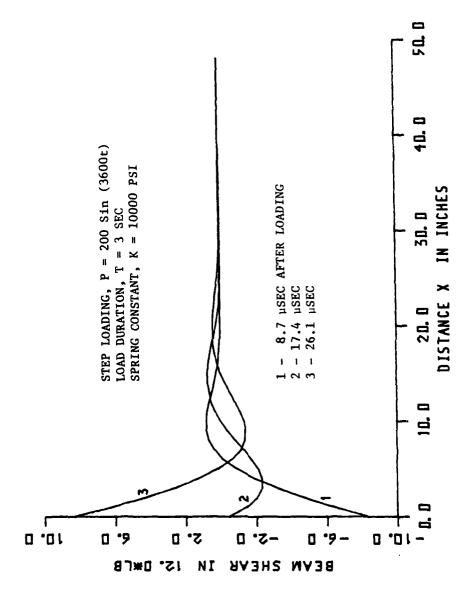


Figure 30. Variation of Moment with Distance for Three Points of Time After Load Application.



Variation of Shear with Distance for Three Points of Time After Load Application. Figure 31.

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4. INFINITE PLATE ON ELASTIC FOUNDATION

4.1 Development of Computer Program

The problem of infinite plate on elastic foundation with vertical loading is axisymmetrical. For this reason, in the development of the computer program, the plate is divided into cells or elements having the shape of concentric rings. As for the beam, the equations of motion and constitutive equations of each cell are derived for both rotation and translation motions. Also, as before, the elastic foundation is assumed to behave as Winker medium.

In the development of physical laws, we consider the free-body diagram of a typical jth element shown in Figure 32 and apply the impulse-momentum laws. The equations of motion thus obtained are as follows:

Rotation of the jth element

$$\frac{d\omega}{dt} = \frac{M_r^{j+1}(r^{j}+dr) - M_r^{j}r^{j} - M_{\theta}^{j} dr - \frac{1}{2}[r^{j}Q_r^{j} dr + (r^{j}+dr)Q_r^{j+1}dr]}{(\rho h^3/12)[r^{j}dr + (dr)^2/2]}$$
(21)

Translation of the jth element

$$\frac{dv}{dt} = \frac{(r^{j} + dr) Q_{r}^{j+1} - r^{j}Q_{r}^{j} - (r^{j} + dr/2) R^{j}}{\rho h [r^{j} dr + (dr)^{2}/2]}$$
(22)

where the foundation reaction R^{j} has been defined in Equations (3) and (4).

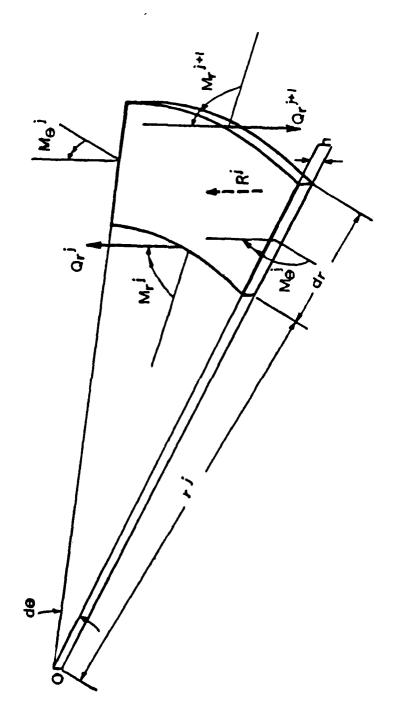


Figure 32. Free-Body Diagram of Plate Element (typical j element)

The constitutive equations are obtained in a manner similar to those of the beam, viz

$$M_{r} = D\{\epsilon_{\phi} + \frac{v}{r} \phi\}$$
 (23)

$$M_{\theta} = D\{\frac{1}{r} \phi + \nu \epsilon_{\phi}\}$$
 (24)

$$Q_r = K_2^2 Gh \{\phi + \epsilon_w\}$$
 (25)

where

$$D = \frac{Eh^3}{12 (1-v^2)}$$
 (26)

$$\varepsilon_{\phi} = (\phi^{j+1} - \phi^{j})/dr \tag{27}$$

$$\varepsilon_{\mathbf{w}} = (\mathbf{w}^{\mathbf{j}+1} - \mathbf{w}^{\mathbf{j}})/\mathrm{dr} \tag{28}$$

$$K_2^2 = 0.76 + 0.3 v$$
 (29)

As for the beam, the dilatation wave velocity is the velocity of propagation of discontinuities in the moments as well as the angular velocity and/or their higher derivatives, and the shear wave velocity is the velocity of propagation of discontinuities in the shear and linear velocity and/or their higher derivatives. These two velocities can be expressed as follows:

Dilatation wave velocity or plate velocity

$$C_{\mathbf{p}} = \left[\frac{E}{\rho (1-v^2)}\right]^{\frac{1}{2}} \tag{30}$$

Shear wave velocity

$$\kappa_2 c_2^{\ 1} = \kappa_2 \left[\frac{G}{O} \right]^{\frac{1}{2}}$$
 (31)

The boundary conditions for the infinite plate on elastic foundation may be derived from the shape of the deflected plate. Under a vertical load uniformly distributed over a small area, the deformed plate resembles a bowl which is axisymmetric about the loading axis so that at the loading center the slope equals zero. Furthermore, according to Alpan and Leshchinsky [18], the plate deflections beyond a distance of R measured from the loading center are very small and can be neglected for practical purposes. The distance R is a function of the radius of the loaded area and the stiffness of the system and can be estimated from the following equation:

$$R = \ell f(r_0/\ell)$$
 (32)

where

r = radius of loaded area

$$\ell$$
 = characteristic length = $4\sqrt{D/k}$

Other notations have been defined earlier. Also, available solutions [19,20] indicate that the deflected shape of the plate is oscillatory and has its first two zeros at 3.92 ℓ and 8.36 ℓ . In this analysis, the larger value of 8.36 ℓ and that computed from Eq. (32) is used to determine the radius of the plate.

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With the use of this distance, the infinite plate may be approximated by a circular plate having a radius equal to R and the loading acting at the center of the plate. Thus, for the problem under consideration, the shear input and boundary conditions are

Shear
$$V]_{x=0} = -\frac{P}{2\pi r_0}$$
 (34)

Slope
$$\psi_{t}]_{x=0} = \omega]_{x=0} = 0$$
 (35)

Deflection
$$y_t]_{x=L} = v]_{x=L} = 0$$
 (36)

where ${\bf r}_{o}$ is the radius of the loaded area, ω and ${\bf v}$ are angular and linear velocities, respectively, and t denotes time.

Using the preceding physical laws, constitutive equations, boundary conditions and the damping equations described in an earlier chapter, the computer program developed by Koenig [16] for circular plates with clamped edges are modified and extended to suit the condition of infinite plates on elastic foundation. The final computer program is included in Appendix B.

4.2 Response to Step Sustained Loading

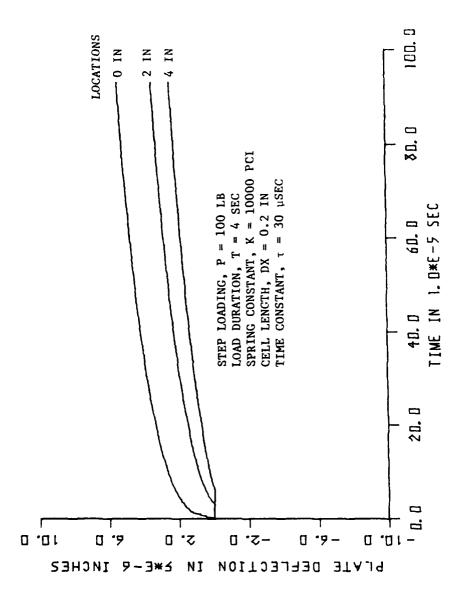
The developed computer program is used to analyze the response of a 4-in. thick infinite plate on elastic foundation to a step pulse loading,

the loading has a magnitude of 100 lb. and is uniformly distributed over a circular area 0.2 in. in radius. The duration of the loading is 4 sec. which is long enough to be considered as a sustained loading. The plate has a Young's modulus of 3 x 10⁶ psi and a Poisson's ratio of 0.20. The supporting foundation is a weightless linear spring having a spring constant of 1 x 10⁴ pci. In the analysis, the 4-in. thick infinite plate is approximated by a circular plate with a radius of 50 in. A cell length of 0.2 in. is used and a time constant of 30 usec is selected in order to obtain an over-damped response.

The analyzed deflection, moment and shear are plotted against time in Figures 33, 34 and 35, respectively. As would be expected, the response at a point increases with increasing time. In general, the shape of the curves resembles that of the curves for beam on elastic foundation.

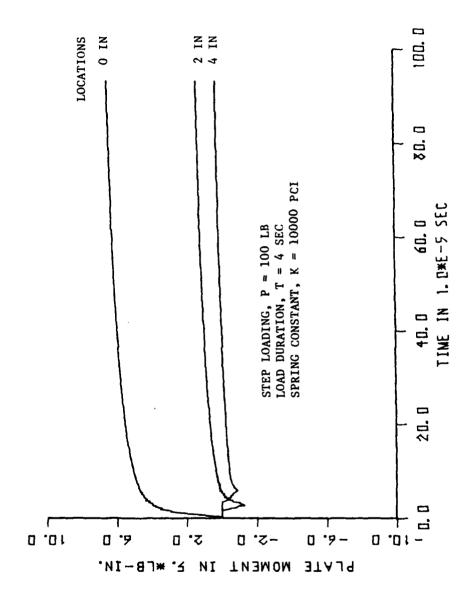
4.3 Response to Sinusoidal Loading

An analysis is also made for the response of the same plate to a sinusoidal vertical loading. The intensity of the loading is also 100 lb., the frequency of excitation is 36000 t. Other conditions including plate modulus, Poisson's ratio, spring constant, cell length and time constant are the same as that used for the beam analyzed previously. Results of the analysis are presented in Figures 36 (plate deflection), 37 (moment) and 38 (shear).



Variation of Plate Deflection with Time for Three Locations Measured from Loading Point, Spring Constant = 10^4 psi. Figure 33.

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Variation of Plate Moment with Time for Three Locations Measured from Loading Point, Spring Constant = $10^4~\rm psi$. Figure 34.

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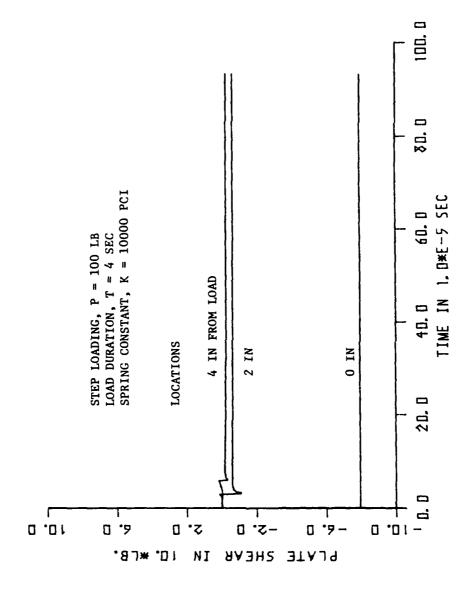
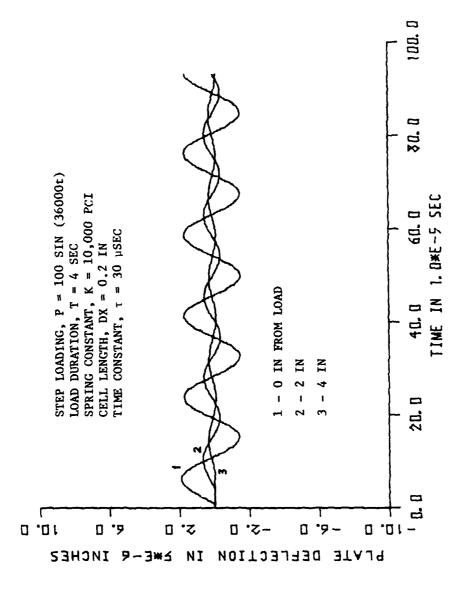
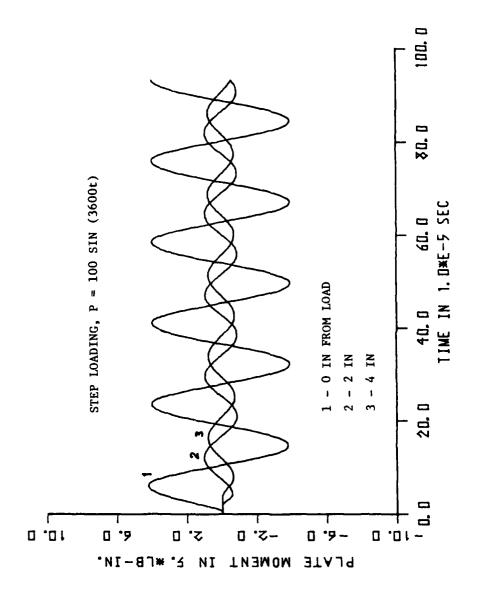


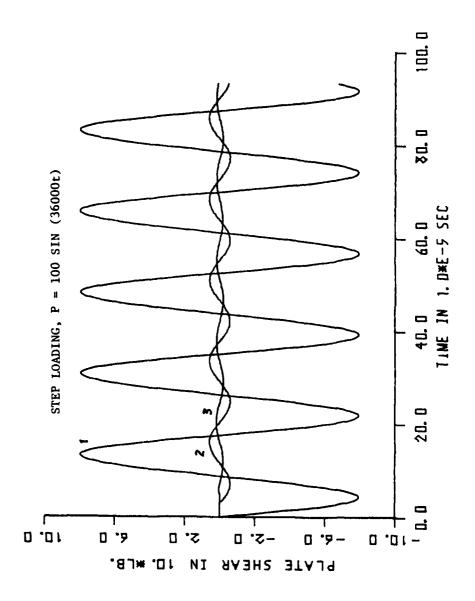
Figure 35. Variation of Plate Shear with Time for Three Locations Measured from Loading Point, Spring Constant = $10^4~\rm psi.$



Variation of Plate Deflection with Time for Three Locations Measured from Loading Point, Spring Constant = 10^4 psi. Figure 36.



Variation of Plate Moment with Time for Three Locations Measured from Loading Point. Figure 37.



Variation of Plate Shear with Time for Three Locations Measured from Loading Point, Spring Constant = $10^4~\rm psi$. Figure 38.

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5. SUMMARY AND CONCLUSIONS

As a step toward the development of a computer program for analyzing the response of elastic layered systems to dynamic loading, the Method of Direct Analysis was used to analyze the dynamic response of infinite beams and plates on elastic foundation. The Method of Direct Analysis uses only the impulse-momentum laws and constitutive relations but bypasses the explicit use of differential equations. As a result of the research, two computer programs were developed, one for infinite beams and the other for infinite plates both on elastic foundations. The computer programs are capable of handling various types of dynamic loading such as sinusoidal, impulse, and sustained loadings. Also, both linear and nonlinear spring supports can be considered.

To demonstrate the effectiveness of the developed computer programs, response analyses were made for sustained, pulse and sinusoidal loadings. For the sustained loading on an infinite beam, both over-damped and under-damped conditions were analyzed. Also, under the pulse loading, the beam was analyzed for nonlinear spring support. The results for sustained loading on an infinite beam were compared with an available exact solution. An excellent agreement with the exact solution was obtained.

The results of the study indicate that the Method of Direct Analysis is an effective tool for dynamic response analysis. It appears feasible that a computer program for response analysis of elastic layered systems

to dynamic loading can be developed based on the principle of the Method of Direct Analysis.

6. RECOMMENDATIONS FOR FURTHER RESEARCH

The computer programs developed from this research are based on the assumption that the elastic foundation is a weightless Winkler medium. Therefore, further research is needed to extend and modify the computer programs to suit elastic layered systems. To achieve the ultimate goal of developing a program for nondestructive pavement testing (NDPT), the following steps of research are recommended:

- The dynamic response of a composite plate consisting of two different materials and supported by a Winkler medium should be studied. Results of the analytical investigation should be validated experimentally.
- 2. Results of the study on composite plate should be extended to the condition of an infinite plate supported by an isotropic elastic half-space. Field testing should be conducted to validate the computer program thus developed.
- 3. Extend the computer program developed above to the condition of a composite plate on an isotropic elastic half-space. This phase of research should also be accompanied by full-scale field testing.

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APPENDIX A

Computer Program for Infinite Beam on Elastic Foundation

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      VOL=REF=MEN.Pl1850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT1, DISP=(OLD, DELETE),
      VOL=REF=MEN.Pl1850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT2, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT3, DISP=(OLD, DELETE),
      VOL=REF=MEN.Pl1850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT4, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT5,DISP=(OLD,DELETE),
      VOL=REF=MEN.Pl1850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT6, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
// EXEC FGCLG.PARM= NOSOURCE
//SYSIN DD *
      REAL YB(900), XB(900)
      LOGICAL*1 SYMA/~"~/,SYMB/~,~/,SYMC/~.~/
      REAL*4 D(6,600)
      INTEGER*2 A(600), B(600), C(600)
      LOGICAL*1 LABEL(40,2)
************
       INFINITE BEAM ON ELASTIC FOUNDATION
*************
C SHEAR AND FLEXURAL WAVES BASED ON DYNAMIC LAWS
  UNITS IN LB-IN-SEC SYSTEM
 POSITIVE SIGN CONVENTION LISTED AFTER DEFINITION
C
С
          = ANGULAR IMPULSE DUE TO MOMENT
  AIM
                                                         LB-IN-SEC.
C
  AIN
          - AREA CONTRIBUTING TO DYNAMIC INERTIA
                                                         IN**2
С
          = ANGULAR IMPULSE DUE TO SHEAR
 AIV
                                                         LB-IN-SEC.
                                                         IN**2
C
          ≖ EFFECTIVE SHEAR CARRYING AREA
  AS
          - DILATATION WAVE VELOCITY
C
  Cl
                                                         IN/SEC.
С
  C2
          = SHEAR WAVE VELOCITY
                                                         IN/SEC.
С
  DE N
          = WEIGHT DENSITY OF BEAM MATERIAL
                                                         LB/CU IN.
C
  DT
          = TIME INCREMENT
                                                         SEC.
C
  DS
          - DISTANCE SCALE FACTOR
                                                          IN.
                                                          SEC.
C
  DTD
          * TIME INCREMENT-DILATATIONAL WAVE
                                                          SEC.
C
  DTR
          ■ TIME INCREMENT-SHEAR WAVE
C
  DX
          = CELL LENGTH
                                                         IN.
          - YOUNG'S MODULUS OF ELASTICITY
                                                          PSI.
C
  EM
C
  ESR
          - FOUNDATION SPRING CONSTANT
                                                         PSI.
C
          = PRINT FORMAT FOR X-T DIAGRAMS
  FMT
          - SHEAR MODULUS OF RIGIDITY
                                                          PSI.
C
  G
C
  IDENT
          = TITLE OF RUN
C
  TM
          - NO. OF CELLS INTO WHICH BEAM IS DIVIDED
C
  INI
          - MOMENT OF INERTIA WHICH CONTRIBUTES RESISTANCE
C
C
  INB
          - MOMENT OF INERTIA WHICH CONTRIBUTES RESISTANCE
                                                          IN**4
            TO DYNAMIC INERTIA
C
          = 0 PLOT THE DATA
C
  TPP
          = 1 PRINT THE DATA
C
  IPP
  IPP
          = 2 PRINT AND PLOT THE DATA
```

```
= NO. OF COLUMNS OF PRINTOUT IN A PAGE
C IPR
           = TIME CONTROL INDEX FOR THE FIRST RESPONSE-DIST. CURVE
C KKM
   KKM1
           = TIME CONTROL INDEX FOR THE SECOND RESPONSE-DIST. CURVE
           ⇒ TIME CONTROL INDEX FOR THE THIRD RESPONSE-DIST. CURVE
  KKM2
           = TIME CONTROL INDEX FOR TERMINATING COMPUTATION
  KM
C
           = SHEAR CORRECTION FACTOR
  KS
C
           = LENGTH OF BEAM
  L
           = LINEAR IMPULSE DUE TO FOUNDATION REACTION
Ç
  LIQ
                                                            LB-SEC.
           = LINEAR IMPULSE DUE TO DISTRIBUTED LOAD
   LIQQ
                                                            LB-SEC.
           - LINEAR IMPULSE DUE TO SHEAR
C
  LIV
                                                            LB-SEC.
  MM
           = SCALED VALUE OF MOMENT
   MOM
           = INTERNAL BENDING MOMENT - CW ON LEFT
                                                            IN-LB.
           = MOMENT SCALE FACTOR
C
  MS
                                                            IN-LB.
C
  MO
           = GIVEN APPLIED MOMENT AT X=0
                                                            IN-LB.
C
   NAME
           = NAME OF MATERIAL OF BODY
C
  NU
           = POISSON'S RATIO
C
  OMEGA
           = VELOCITY OF ROTATION OF CROSS-SECTION,
C
             D(PSI)/DT - CLOCKWISE
                                                            RAD/SEC.
С
   OMGS
           = ANGULAR VELOCITY SCALE FACTOR
                                                            RAD/SEC.
C
  MOMO
           = SCALED VALUE OF ANGULAR VELOCITY
C
           = ROTATION OF CROSS-SECTION ABOUT THE NEUTRAL
C
             AXIS - CW ON LEFT
                                                                 RAD.
C
  PSIPS
           = SCALED VALUE OF ROTATION
  PSIS
           = ROTATION SCALE FACTOR
                                                            RAD.
C
C
           = INTENSITY OF UNIFORMLY DISTRIBUTED EXTERNAL
             LOADING - DOWNWARD
                                                            LB/IN.
C
C
   QQ0
           = INPUT VALUE OF DISTRIBUTED LOAD
                                                            LB/IN.
C
   RHO
           = MASS DENSITY OF BEAM MATERIAL
                                                         LB-SEC**2/IN**
C
   T
           = TIME
                                                            SEC.
C
   TAU1
           = DAMPING TIME CONSTANT FOR OMEGA
                                                            SEC.
C
  TAU2
           = DAMPING TIME CONSTANT FOR VELOCITY
                                                            SEC.
C
  TD
           = TIME CLOCK WHICH REGULATES DILATATIONAL WAVE SEC.
           = THE PREVIOUS TD OR TR
С
  TL
                                                            SEC.
C
  TLB
           = ONE TIME SEGMENT BEFORE TL
                                                            SEC.
С
  TR
           - TIME CLOCK WHICH REGULATES SHEAR WAVE
                                                            SEC.
C
  TS
           = TIME SCALE FACTOR
С
 TT
           = SCALED VALUE OF TIME
С
  T1
           = DURATION OF MOMENT PULSE AT X=0
           = DURATION OF VELOCITY PULSE AT X=0
С
  Т3
           = VERTICAL SHEAR ON A CROSS-SECTION -UP ON LEFT LB.
  VEL
           = VELOCITY OF DEFLECTION, D(Y)/DT - DOWNWARD
                                                            IN/SEC.
  VELS
           = LINEAR VELOCITY SCALE FACTOR
                                                            IN/SEC.
           = SCALED VALUE OF LINEAR VELOCITY
  VELVE
С
  ٧S
           = SHEAR SCALE FACTOR
                                                            LB.
C
  VV
           = SCALED VALUE OF SHEAR
C
  40
           = GIVEN APPLIED SHEAR AT X=0
C
           = AXIAL COORDINATE ALONG LENGTH OF BEAM MEASURED
             FROM LEFT END
C
                                                            IN.
C
           = DEFLECTION - DOWNWARD
                                                            IN.
           = DEFLECTION SCALE FACTOR
C
  YS
                                                            IN.
           - SCALED VALUE OF DEFLECTION
С
C
C
             MAIN PROGRAM
```

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DIMENSIONX(400), V(400), VEL(400), Y(400), PSI(400), OMEGA(400), VV(400)

```
1, VELVE(400), OMOM(400), YY(400), PSIPS(400), Q(400), YB1(900), YB2(900)
     REALINB, INI, L, MOM(400), MO, MS, MM(400), NU, KS, MASS, INERT
     LOGICAL*11DENT(30), FMT(80), NAME(10)
     COMMONT, X, G, EM, L, MOM, V, C1, C2, Q, VEL, Y, RHO, DEN, PSI, OMEGA, INB, INI, AIN
    1, AS, KS, MO, VO, T1, T3, DT, DX, TS, MS, VS, VELS, OMGS, TT, VV, MM, VELVE, OMOM, IM
    1, KM, IPR, MASS, INERT, KKM, PSIPS, PSIS, YS, IMO, TAU1, TAU2, QOO, ESR, BETA
    1,KKM1,KKM2
   1 READ 801, IDENT, FMT
     READ 802, NAME, DEN, EM, L, NU, DX, AIN, KS
     IF(DEN) 500,500,3
   3 READ 803, MO, VO, T1, T3, TS, VS, VELS, YS
     READ 804, KM, IPR, INB, MS, PSIS, OMGS, TAU1, TAU2
     READ 805, ESR, KKM, KKM1, KKM2, IPP, US
     G=EM/(2.0*(1.0+NU))
     RHO=DEN/386.0
     AS=KS*AIN
     INI=INB
     C1=(((EM*INB)/(RHO*INI))**0.5)
     C2=(((AS*G)/(RHO*AIN))**0.5)
     MASS=RHO*AIN*DX
     INERT=RHO*INI*DX
     BETA=(ESR/(4.0*EM*INB))**0.25
     DT=DX/(C1)
     GAMMA=EM/(G*KS)
     DIVISION OF BEAM INTO ELEMENTS
     X(1)=0.0
     DO 101 I=1,400
     X(I+1)=X(I)+DX
     IF(ABS(X(I+1)-L)-DX/2.0) 10,10,101
10
     IM=I
     GO TO 11
101 CONTINUE
     IM0=IM+1
     PRINT 900, IDENT
     PRINT 901, NAME, DEN, RHO, EM, G, ESR, NU
     PRINT 902, L, AS, AIN, INB, INI, TAU1, TAU2
     PRINT 903,C1,C2
     PRINT 904, MO, VO, T1, T3
     PRINT 905, DT, DX, TS, VS, VELS, OMGS, MS, PSIS, YS
     PRINT 906, IM, KM, IPR
     PRINTOUT OF CELL BOUNDARY COORDINATES
     PRINT 9266
     PRINT 907,(I,X(I),I=1,IM0)
     DO 909 M=11,13
909 REWIND M
     DO 910 M=20,22
910 REWIND M
     CALL BEAM
     *****
     PRINT INSTRUCTIONS
     IF(IPP.EQ.0) GO TO 500
     IA=1
     IM2=IPR
  30 DO 912 M=11,13
```

С

C

C

```
912 REWIND M
      DO 913 M=20,22
 913
      REWIND M
      IF(IM+1-IM2) 24,24,25
      IM2=IM+1
      X-T DIAGRAM FOR MOMENT
   25 PRINT 908
      DO 104 K=1,KM
      READ (11) TT,MM(1),VV(1),(MM(I),I=1,IM0)
     PRINT FMT, TT, MM(1), VV(1), (MM(I), I=IA, IM2)
      X-T DIAGRAM FOR SHEAR
      PRINT 1909
      DO 105 K=1,KM
      READ (12) TT, MM(1), VV(1), (VV(I), I=1, IM0)
  105 PRINT FMT, TT, MM(1), VV(1), (VV(I), I=IA, IM2)
      X-T DIAGRAM FOR LINEAR VELOCITY
      PRINT 1910
      DO 106 K=1,KM
      READ (20) TT, MM(1), VV(1), (VELVE(I), I=1, IMO)
  106 PRINT FMT, TT, MM(1), VV(1), (VELVE(I), I=IA, IM2)
С
      X-T DIAGRAM FOR DEFLECTION
      PRINT 911
      DO 107 K=1,KM
      READ (21) TT,MM(1),VV(1),(YY(I),I=1,IM0)
      PRINT FMT, TT, MM(1), VV(1), (YY(I), I=IA, IM2)
 107
      CONTINUE
      X-T DIAGRAM FOR ANGULAR VELOCITY
      PRINT 1912
      DO 108 K=1,KM
      READ (22) TT,MM(1),VV(1),(OMOM(1),I=1,IM0)
  108 PRINT FMT, TT, MM(1), VV(1), (OMOM(I), I=IA, IM2)
      X-T DIAGRAM FOR ANGLE OF ROTATION
      PRINT 4913
      DO 4108 K=1,KM
      READ (13) TT, MM(1), VV(1), (PSIPS(I), I=1, IM0)
 4108 PRINT FMT, TT, MM(1), VV(1), (PSIPS(I), I=IA, IM2)
      IF(IM-IM2) 500,500,37
 37
      IA=IA+IPR
      IM2=IM2+IPR
      GO TO 30
      IF(IPP.EQ.1) GO TO 501
 800 FORMAT(80A1)
 801
      FORMAT(80A1/80A1)
 802
      FORMAT(10A1, F10.2, E10.2, 3F10.2, E10.2, F10.2)
 803
      FORMAT(8E10.2)
 804
      FORMAT(2110,6E10.2)
 805
      FORMAT(E10.2,4110,F10.2)
 900
      FORMAT(1HD,80A1//)
  901
              FORMAT(26HOMATERIAL
                                                                       ,10A1
             ,24HOWEIGHT DENSITY
                                     DEN = ,F8.3,20H LBS/IN**3
     2
             ,24HOMASS DENSITY
                                     RHO = ,F8.6,20H LB/SEC**2/IN**4
     3
             ,24HOYOUNGS MODULUS
                                       E = ,E8.2,20H PSI
             ,24HOMOD. OF RIGIDITY
                                       G = ,E8.2,20H PSI
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73
            ,24HOMOD. OF SUB. REAC.ESR = ,E8.2,29H PSI
                                  NU = ,F8.2,20H
            .24HOPOISSONS RATIO
                                                                      1)
 902 FORMAT(24HOLENGTH OF BEAM
                                    L = ,F8.3,20H IN
                                    AS = ,F8.2,17H IN**2
     1
             24HOSHEAR AREA
                                  AIN = ,F8.2,17H IN**2
             24HOINERTIA AREA
             24HOBEND. MOM. IN.
                                 INB = ,F8.2,17H IN**4
             24HOINERTIA MOM. IN. INI =
                                        ,F8.2,17H IN**4
     5
             24H0 TAU1
                                 TAU1 =
                                        ,E8.2,17H 1/SEC
             24H0 TAU2
                                 TAU2 =
                                                                      1)
                                        ,E8.2,17H 1/SEC
 903 FORMAT(24HOMOMENT WAVE
                                   C1 =
                                         ,F8.0,20H IN/SEC
                                   C2 = ,F8.0,20H IN/SEC
             24HOSHEAR WAVE
                                  MO = F8.2,20H IN-LB
 904 FORMAT(24HOX=0 MOMENT PULSE
             24HOX=0 VELOC PULSE
                                  VO = ,F8.2,20H LB
     2
             24HOX=0 MOM. PULSE DUR.T1=
                                         ,E8.2,20H SEC
             24HOX=0 VEL PULSE DUR.T3 =
                                         ,E8.2,20H SEC
                                   DT =
 905 FORMAT(24HOTIME INCREMENT
                                         ,E8.2,20H SEC
             24HOCELL LENGTH
                                   DX =
                                         ,F8.5,20H IN
     1
     2
             24HOTIME SCALE FACTOR TS =
                                         ,E8.2,20H SEC
    3
             24HOSHEAR SCALE FACTOR VS= ,E8.2,20H LB
             24HOVEL SCALE FACTOR VELS=
                                         ,E8.2,20H IN/SEC
     4
             24HOANG VEL SCALE FAC OMGS=
     5
                                         ,E8.2,20H RAD/SEC
             24HOMOMENT SCALE FACTOR MS= ,E8.2,20H IN-LB
             24HOANG SCALE FACTOR PSIS= ,E8.2,20H RAD
             24HODEF SCALE FACTOR
                                  YS = , E8.2, 20H IN
                                         ,14,17H
 906 FORMAT(24HONO. OF CELLS
                                    IM =
             24HOTOT. TIME INTERVALS KM=
                                         , I4, 17H
             24HONO. COLS. PRINTED IPR= ,14,
                                                                   1111)
 907 FORMAT(14,F8.4)
 908 FORMAT(1H1,2X, TIME, 5X, PULSE, 24X, MOMENT
                                                    PROPAGATION /)
 911 FORMAT(1H1,2X, TIME, 5X, PULSE, 21X, DEFLECTION PROPAGATION)
1909 FORMAT(1H1,2X, TIME, 5X, PULSE, 24X, SHEAR PROPAGATION)
1910 FORMAT(1H1,2X, TIME, 5X, PULSE, 16X, LINEAR VELOCITY PROPAGATION
    1/)
 1912 FORMAT(1H1,2X, TIME',5X, PULSE',15X, ANGULAR VELOCITY PROPAGATION
 4913 FORMAT(1H1,2X, TIME, 5X, PULSE, 23X, ROTATION
                                                      PROPAGATION /)
 9266 FORMAT(1HS, 2X, 'I', 3X, 'X(I)',/)
 500 REWIND 21
      DO 127 K=1,KM
      READ (21) TT,MM(1),VV(1),(YY(I),I=1,IM0)
     XB(K)=TT
      YB(K)=YY(1)
      YB1(K)=YY(4)
      YB2(K)=YY(7)
127 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,30)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET GRPHIT ARGUMENT VALUES.
      N=KM
      LOGX=0
      LOGY=0
      XAXIS=8.0
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YAXIS=5.0
     XMIN=0.0
     NXDEC=5
     XINC=20.0
     YMIN=-10.0
     NYDEC=5
     YINC=4.0
     XSMIN=0.0
     XSMAX=100.0
     YSMIN=-10.0
     YSMAX=10.0
     NDECX=1
     NDECY=1
     HT=0.15
     HTS=0.15
     ICON=0
     CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
    1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
    1 NDECX, NDECY, HTS)
     DO 130 I=1, N
130 YB(I)=YB1(I)
     ICGN=0
     CALL SAMEP(YB, N, XB, SYMB, ICON)
     DO 71 I=1, N
71
     YB(I)=YB2(I)
     ICON=0
     CALL SAMEP(YB, N, XB, SYMC, ICON)
     CALL EPLOT
     REWIND 12
     DO 128 K=1,KM
     READ (12) TT, MM(1), VV(1), (VV(I), I=1, IMO)
     XB(K)=TT
     YB(K)=VV(1)
     YB1(K)=VV(4)
     YB2(K)=VV(7)
128 CONTINUE
     CALL INITQ(A,B,C,D,600)
     CALL STSWQ(4662,31)
     READ(5,800) (LABEL(J,1),J=1,40)
     READ(5,800) (LABEL(J,2),J=1,40)
     SET GRPHIT ARGUMENT VALUES.
     N=KM
     LOGX=0
     LOGY=0
     XAXIS=8.0
     YAXIS=5.0
     XMIN=0.0
     NXDEC=5
     XINC=20.0
     YMIN=-10.0
     NYDEC=5
     YINC=4.0
     XSMIN=0.0
     XSMAX=100.0
```

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YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      MIS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 72 I=1, N
72
     YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 73 I=1,N
 73
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 11
      DO 129 K=1,KM
      READ (11) TT, MM(1), VV(1), (MM(I), I=1, IMO)
      XB(K)=TT
      YB(K)=MM(1)
      YB1(K)=MM(4)
      YB2(K)=MM(7)
 129 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,32)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
С
      SET GRPHIT ARGUMENT VALUES.
      N=KM
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=20.0
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=100.0
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
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1 NDECX, NDECY, HTS)
      DO 74 I=1, N
  74 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 75 I=1,N
 75
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 21
      DO 131 K=1,KM
      READ (21) TT,MM(1),VV(1),(YY(I),I=1,IM0)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 131
      DO 231 I=1, IMO
      XB(I)=X(I)
      IF(K.EQ.KKM) YB(I)=YY(I)
      IF(K.EQ.KKM1) YB1(I)=YY(I)
      IF(K.EQ.KKM2) YB2(I)=YY(I)
 231 CONTINUE
 131 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,33)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET GRPHIT ARGUMENT VALUES.
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 76 I=1, N
 76 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 77 I=1,N
```

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77
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 12
      DO 132 K=1.KM
      READ (12) TT, MM(1), VV(1), (VV(I), I=1, IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 132
      DO 232 I=1, IMO
      XB(I)=X(I)
      IF(K.EQ.KKM) YB(I)=VV(I)
      IF(K.EQ.KKM1) YB1(I)=VY(I)
      IF(K.EQ.KKM2) YB2(I)=VV(I)
232 CONTINUE
132 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,34)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET GRPHIT ARGUMENT VALUES.
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 78 I=1.N
 78 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 79 I=1.N
79
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 11
      DO 133 K=1,KM
```

```
READ (11) TT,MM(1),VV(1),(MM(I),I=1,IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 133
      DO 233 I=1, IMO
      XB(I)=X(I)
      IF(K.EQ.KKM) YB(I)=MM(I)
      IF(K.EQ.KKM1) YB1(I)=MM(I)
      IF(K.EQ.KKM2) YB2(I)=MM(I)
233 CONTINUE
133 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,35)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET GRPHIT ARGUMENT VALUES.
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 81 I=1,N
  81 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 82 I=1,N
82
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      YB(1)=0.0
      YB1(1)=0.0
      YB2(1)=0.0
      REWIND 21
      DO 136 K=1,KM
      READ (21) TT,MM(1),VV(1),(YY(1),I=1,IM0)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 136
      DO 236 I=1.IM
```

```
XB(I)=X(I)
      IF(K.EQ.KKM) YB(I+1)=(YY(I+1)-YY(I))/DX
      IF(K.EQ.KKM1) YB1(I+1)=(YY(I+1)-YY(I))/DX
      IF(K.EQ.KKM2) YB2(I+1)=(YY(I+1)-YY(I))/DX
 236
      CONTINUE
136 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,36)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
      SET GRPHIT ARGUMENT VALUES.
C
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-1.0
      NYDEC=5
      YINC=0.4
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-1.0
      YSMAX=1.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 84 I=1,N
  84 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 85 I=1,N
 85
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
 501
      STOP
      E ND
                                SUBROUTINE BEAM
      DIMENSIONX(400), V(400), VEL(400), Y(400), PSI(400), OMEGA(400), VV(400)
     1, VELVE(400), OMOM(400), YY(400), PSIPS(400), AIM(400), AIV(400), Q(400)
      REALINB, INI, L, MOM(400), MO, MS, MM(400), NU, KS, MASS, INERT, LIV (400), LIQ
     1(400)
      LOGICAL*11DENT(80), FMT(80), NAME(10)
      COMMONT, X, G, EM, L, MOM, V, C1, C2, Q, VEL, Y, RHO, DEN, PSI, OMEGA, INB, INI, AIN
     1, AS, KS, MO, VO, T1, T3, DT, DX, T5, MS, VS, VELS, OMGS, TT, VV, MM, VELVE, OMOM, IM
     1, KM, IPR, MASS, INERT, KKM, PSIPS, PSIS, YS, IMO, TAU1, TAU2, QQO, ESR, BETA
```

```
1,KKM1,KKM2
      PRINT 921
      INITIAL STATE OF BODY
C
      DO 119 I=1, IMO
      AIM(I)=0.0
      AIV(I)=0.0
      LIV(I)=0.0
      LIQ(I)=0.0
      LIQQ(I)=0.0
      MOM(I)=0.0
      OMEGA(I)=0.0
      PSI(I)=0.0
     V(I)=0.0
     VEL(I)=0.0
 119 Y(I)=0.0
      QQ=QQO
      DTD=DX/(C1)
      DTR=DX/(C2)
      T=0.0
      TD=0.0
      TL=0.0
      TLB=0.0
      TR=0.0
      DO 101 K=1,KM
      TT=T/TS
      ***INPUT AT X=0, MOMENT BOUNDARY CONDITION***
      IF(T-T1) 110,111,111
110 MOM(IMO)=MO
      GO TO 1492
111 MOM(IMO)=0.0
1492 CONTINUE
      ***INPUT AT X=0, SHEAR BOUNDARY CONDITION***
      IF(T-T3) 113,114,114
113 V(1)=V0
      GO TO 115
114 V(1)=0.0
115 CONTINUE
      ***REACTION OF ELASTIC FOUNDATION***
      DO 116 I=1, IM
      Q(I) = -ESR*Y(I)/(1+ESR*Y(I)/100.0)
116
     CONTINUE
      SCALING FOR PRINTOUT
      IZZ=IMO
      DO 103 I=1, IZZ
     MM(I)=MOM(I)/MS
      OMOM(I)=OMEGA(I)/OMGS
      PSIPS(I)=PSI(I)/PSIS
     VELVE(I)=VEL(I)/VELS
     VV(I)=V(I)/VS
      YY(1)=Y(1)/YS
103
     CONTINUE
     WRITE (11) TT,MM(1),VV(1),(MM(1),I=1,IMO)
     WRITE (12) TT,MM(1),VV(1),(VV(I),I=1,IMO)
      WRITE (13) TT,MM(1),VV(1),(PSIPS(I),I=1,IMO)
```

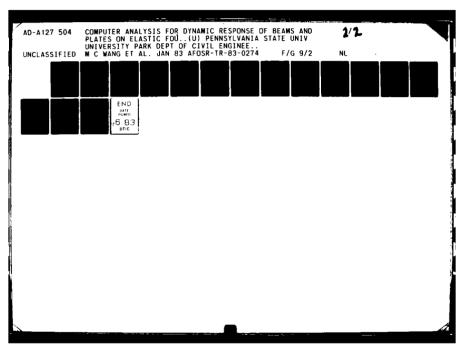
```
WRITE (20) TT,MM(1),VV(1),(VEIVE(I),I=1,IM0)
      WRITE (21) TT, MM(1), VV(1), (YY(I), I=1, IMO)
      WRITE (22) TT,MM(1),VV(1),(OMOM(I),I=1,IMO)
      IF(ABS(TD-(T+DT))-0.1E-10) 19,19,89
 12
89
      IF(TD-TR) 15,15,16
 16
      IF(TD+DTD-TR-DTR) 15,15,13
      PROPAGATION PROCEDURE
С
      THE C1 WAVE
15
      DO 108 I=2, IMO
C
      ANGULAR IMPULSE DUE TO MOMENT
      AIM(I)=(MOM(I-1)-MOM(I))*DTD+AIM(I)
C
      ANGULAR IMPULSE DUE TO SHEAR
      AIV(I)=(V(I-1)+V(I))*DX*(TD+DTD-TL)/2.0+AIV(I)
C
      CUMULATE LINEAR IMPULSE DUE TO THIS SHEAR
      LIV (I-1)=LIV (I-1)+(V(I)-V(I-1))*(TD+DTD-TL)
C
      DAMPING, FIRST HALF
      IF(TAU1-1.0E+40) 930,928,928
930 DOMG=(-1.0)*OMEGA(I)*(DTD/TAU1)
      OMEGA(I) = OMEGA(I) + (DOMG*0.5)
      IMPULSE-MOMENTUM ACROSS C1
928 DOM=(AIM(I)+AIV(I))/INERT
      OMEGA(I)=OMEGA(I)+DOM
C
      BOUNDARY CONDITION FOR OMEGA
      OMEGA(1)=0.0
      PSI(1)=0.0
      DAMPING, SECOND HALF
C
      IF(TAU1-1.0E+40) 9113,815,815
9113 DOMG=(-1.0)*OMEGA(I)*(DTD/TAUL)
      OMEGA(I) = OMEGA(I) + (DOMG*0.5)
      INITIALIZE ANGULAR IMPULSES AFTER USE
 815 AIV(I)=0.0
      AIM(I)=0.0
 108 CONTINUE
      TD=TD+DTD
      TLB=TL
      TL=TD
      DO 107 I=1, IM
      DALPH=(OMEGA(I+1)-OMEGA(I))*DTD)/DX
C
      CONSTITUTIVE EQUATION FOR INCREMENTAL MOMENT
      DMOM=(-1.0)*EM*INB*DALPH
C
      CUMULATE MOMENT
      MOM(I)=MOM(I)+DMOM
C
      CUMULATE SHEAR (ENTIRE OMEGA PORTION)
     V(I+1)=V(I+1)-AS*G*OMEGA(I+1)*DTD
C
      ROTATION
      PSI(I+1)=PSI(I+1)+(OMEGA(I+1)*DTD)
 107
     CONTINUE
      GO TO 12
      THE C2 WAVE
      DO 104 I=1, IM
13
C
      LINEAR IMPULSE DUE TO FOUNDATION REACTION
      LIQ(I)=Q(I)*DX*DTR+LIQ(I)
C
      LINEAR IMPULSE DUE TO SHEAR
      LIV(I)=(V(I+1)-V(I))*(TR+DTR-TL)+LIV(I)
```

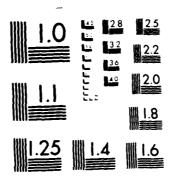
```
CUMULATING ANGULAR IMPULSE DUE TO SAME SHEAR
C
      AIV (I+1)=AIV (I+1)+0.5*(V (I)+V (I+1))*DX*(TR+DTR-TL)
C
      DAMPING, FIRST HALF
      IF(TAU2-1.0E+40) 925,931,931
 925 DVELG=(-1.0)*VEL(I)*(DTR/TAU2)
      VEL(I)=VEL(I)+(DVELG*0.5)
      IMPULSE-MOMENTUM ACROSS C2
 931 DVEL=(LIV(I)+LIQ(I))/MASS
      VEL(I)=VEL(I)+DVEL
      BOUNDARY CONDITION FOR VELOCITY
      VEL(IM+1)=0.0
      DAMPING, SECOND HALF
      IF(TAU2-1.0E+40) 927,818,818
 927 DVELG=(-1.0)*VEL(I)*(DTR/TAU2)
      VEL(I)=VEL(I)+(DVELG*0.5)
      INITIALIZING LINEAR IMPULSES AFTER USE
 818 LIV(I)=0.0
      LIQ(I)=0.0
 104 CONTINUE
      TR=TR+DTR
      TLB=TL
      TL=TR
      DO 102 I=1, IM
С
      LINEAR STRAIN
      DSTRN=((VEL(I+1)-VEL(I))*DTR)/DX
C
      CONSTITUTIVE EQUATION FOR INCREMENTAL SHEAR (VELOCITY PORTION)
      DV =AS*G*DSTRN
C
      CUMULATE SHEAR
      V(I+1)=V(I+1)+DV
C
      DEFLECTION
      Y(I)=Y(I)+(VEL(I)*DTR)
 102 CONTINUE
      GO TO 12
 19
      CONTINUE
      T=T+DT
 101 CONTINUE
      ENDFILE 11
      ENDFILE 12
      ENDFILE 13
      ENDFILE 20
      ENDFILE 21
      ENDFILE 22
 921 FORMAT(1H1)
 500
     RETURN
//DATA.FT11F001 DD UNIT=SYSDA, SPACE=(TRK, (20,10), RLSE)
//DATA.FT12F001 DD UNIT=SYSDA, SPACE=(TRK, (20,10), RLSE)
//DATA.FT13F001 DD UNIT=SYSDA, SPACE=(TRK, (20,10), RLSE)
//DATA.FT20F001 DD UNIT=SYSDA, SPACE=(TRK, (20,10), RLSE)
//DATA.FT21F001 DD UNIT=SYSDA, SPACE=(TRK, (20, 10), RLSE)
//DATA.FT22F001 DD UNIT=SYSDA, SPACE=(TRK, (20, 10), RLSE)
//DATA.INPUT DD *
BOLEY CHECK
(1HS,F6.2,1X,F5.1,1X,F5.1,1X,22F5.2)
```

```
30.00E+5
              0.0868
                                     48.00
                                                0.20
                                                            1.00
                                                                    4.0E+0
 CONCRETE
0.833
   0.00E+0 -100.0E+0
                        3.00E+0
                                   3.00E+0
                                              1.00E-4
                                                        12.00E+0
                                                                   1.00E-2
1.50E-4
                         5.33E+0
                                  50.00E+0
                                             1.00E-5
                                                         1.00E-2
                                                                   3.00E-5
       900
3.00E-5
                  50
                            100
                                              50.00
   1.00E+4
                                      150
TIME IN 1.0*E-4 SEC
BEAM DEFLECTION IN 1.5*E-4 INCHES
TIME IN 1.0*E-4 SEC
BEAM SHEAR IN 12.0*LB.
TIME IN 1.0*E-4 SEC
BEAM MOMENT IN 50.0*LB-IN.
DISTANCE X IN INCHES
BEAM DEFLECTION IN 1.5*E-4 INCHES
DISTANCE X IN INCHES
BEAM SHEAR IN 12.0*LB
DISTANCE X IN INCHES
BEAM MOMENT IN 50.0*LB-IN.
DISTANCE X IN INCHES
BEAM SLOPE IN 1.5*E-4
//DATA.FT30F001 DD DSN=MEN.P11850.A3B.PLOT,
11
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
11
               SPACE=(TRK,(20,2),RLSE),
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT31F001 DD DSN=MEN.P11850.A3B.PLOT1,
11
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP).
               SPACE=(TRK,(20,2),RLSE),
11
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT32F001 DD DSN=MEN.P11850.A3B.PLOT2,
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
//
               SPACE=(TRK,(20,2),RLSE),
//
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT33F001 DD DSN=MEN.P11850.A3B.PLOT3,
               VOL=REF=MEN.P11850.A3B.LIB.DISP=(NEW, KEEP),
//
               SPACE=(TRK,(20,2),RLSE),
//
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT34F001 DD DSN=MEN.P11850.A3B.PLOT4,
//
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
11
               SPACE=(TRK,(20,2),RLSE)
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT35F001 DD DSN=MEN.P11850.A3B.PLOT5,
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
//
//
               SPACE=(TRK,(20,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT36F001 DD DSN=MEN.P11850.A3B.PLOT6,
//
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
//
               SPACE=(TRK,(20,2),RLSE)
//
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
```

APPENDIX B

Computer Program for Infinite Plate on Elastic Foundation





MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS (963.4

```
// EXEC PGM=IEFBR14
//D DD DSN=MEN.P11850.A3B.PLOT, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT1, DISP=(OLD, DELETE),
      YOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.P11850.A3B.PLOT2, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//
//D DD DSN=MEN.P11850.A3B.PLOT3, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//
//D DD DSN=MEN.P11850.A3B.PLOT4, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.Pl1850.A3B.PLOT5, DISP=(OLD, DELETE),
      VOL=REF=MEN.P11850.A3B.LIB
//D DD DSN=MEN.Pl1850.A3B.PLOT6, DISP=(OLD, DELETE),
    VOL=REF=MEN.P11850.A3B.LIB
// EXEC FGCLG, PARM= NOSOURCE
//SYSIN DD *
*****************
       CIRCULAR PLATE ON ELASTIC FOUNDATION
**********************
  SHEAR AND FLEXURAL WAVES BASED ON DYNAMIC LAWS
  UNITS IN LB-IN-SEC SYSTEM
С
          - ANGULAR IMPULSE DUE TO RADIAL MOMENT
 AIMR
                                                        LB-IN-SEC.
С
С
 AIMTH
         - ANGULAR IMPULSE DUE TO TANGENTIAL MOMENT
                                                         LB-IN-SEC.
C
          = ANGULAR IMPULSE DUE TO TRANSVERSE SHEAR
                                                         LB-IN-SEC.
 AIQR
C
 ALPHA
          = ANGULAR STRAIN - D(PHI)/DR
                                                         RAD/IN.
C CP
          - PLATE VELOCITY
                                                         IN/SEC.
C C2
          = SHEAR WAVE VELOCITY
                                                         IN/SEC.
C D
          FLEXXURAL RIGIDITY
                                                         LB-IN.
C DALPH
          = INCREMENTAL ANGULAR STRAIN
                                                         RAD/IN.
C DEN
          - WEIGHT DENSITY OF THE PLATE MATERIAL
                                                        LB/CU IN.
          = CELL LENGTH
C
 DR
                                                         TN.
С
 DSTRN
         = INCREMENTAL LINEAR STRAIN
С
 DS
          = DISTANCE SCALE FACTOR
                                                         IN.
С
 DT
          = TIME INCREMENT
                                                         SEC.
С
  DTD
          = TIME INCREMENT-DILATATIONAL WAVE
                                                         SEC.
C
  DTR
          = TIME INCREMENT-SHEAR WAVE
                                                         SEC.
          - YOUNG'S MODULUS OF ELASTICITY
  EM
                                                         PSI.
С
  ESR
          = FOUNDATION SPRING CONSTANT
                                                         PSI.
C
  FMT
          = PRINT FORMAT FOR X-T DIAGRAMS
C
  G
          = SHEAR MODULUS OF RIGIDITY
                                                         PSI.
C H
          = THICKNESS OF THE PLATE
                                                         IN.
C
 IDENT
          = TITLE OF RUN
C
 IM
          = NO. OF CELLS INTO WHICH PLATE IS DIVIDED
С
 IPP
          = 0 PLOT THE DATA
C IPP
          = 1 PRINT THE DATA
          = 2 PRINT AND PLOT THE DATA
C IPP
C IPR
          = NO. OF COLUMNS OF PRINTOUT IN A PAGE
C KKM
          = TIME CONTROL INDEX FOR THE FIRST RESPONSE-DIST. CURVE
C KKM1
          - TIME CONTROL INDEX FOR THE SECOND RESPONSE-DIST. CURVE
          = TIME CONTROL INDEX FOR THE THIRD RESPONSE-DIST. CURVE
C
 KKM2
C KM
          - TIME CONTROL INDEX FOR TERMINATING COMPUTATION
С
  K2
          - SHEAR CORRECTION FACTOR
```

The second secon

```
= LINEAR IMPULSE DUE TO TRANSVERSE SHEAR
                                                              LB-SEC.
C
   LIQR
           - SCALED VALUE OF RADIAL MOMENT
   MMR
   MMRO
           = SCALED VALUE OF R=RO RADIAL MOMENT INPUT
                                                              IN-LB.
   MMTH
           = SCALED VALUE OF TANGENTIAL MOMENT
                                                              IN-LB.
           * RADIAL BENDING MOMENT PER UNIT LENGTH
                                                              LB-IN/IN.
C
  MR
C
   MRS
           - RADIAL MOMENT SCALE FACTOR
                                                              LB-IN/IN.
Ç
  MRO
           = GIVEN APPLIED RADIAL MOMENT AT R=0
                                                              IN-LB.
C
  MTH
           = TANGENTIAL BENDING MOMENT PER UNIT LENGTH
                                                              LB-IN/IN.
С
  MTHS
           - TANGENTIAL MOMENT SCALE FACTOR
                                                              LB-IN/IN.
           - NAME OF MATERIAL OF BODY
C
   NAME
C
   NU
           = POISSON'S RATIO
C
   OMEGA
           - VELOCITY OF ROTATION OF CROSS-SECTION,
C
             D(PSI)/DT - CLOCKWISE
                                                              RAD/SEC.
                                                              RAD/SEC.
C
   OMGS
           = ANGULAR VELOCITY SCALE FACTOR
C
           - SCALED VALUE OF ANGULAR VELOCITY
   OMOM
C
           = ROTATION OF THE CROSS-SECTION ABOUT THE
   PHI
C
             TANGENTIAL AXIS
                                                              RAD.
           = SCALED VALUE OF SHEAR
C
   90
           = SCALED VALUE OF R=RO SHEAR INPUT
C
   QQO
C
   OR
           = TRANSVERSE SHEAR PER UNIT LENGTH
                                                              LB/IN.
C
   ORS
           = SHEAR SCALE FACTOR
                                                              LB/IN.
C
   ORO
           = GIVEN APPLIED TRANSVERSE SHEAR AT R≈RO
C
           = RADIAL COORDINATE FROM THE CENTER OF PL
                                                              IN.
   R
           - MASS DENSITY OF PLATE MATERIAL
                                                          LB-SEC**2/IN**4
С
   RHO
C
   RL
           - OUTER RADIUS OF THE PLATE
                                                              IN.
                                                              IN.
C
   RO
           = INNER RADIUS OF THE PLATE
C
   STRAIN = LINEAR STRAIN - D(W)/DR
C
           - TIME
                                                              SEC.
C
           = DAMPING TIME CONSTANT FOR OMEGA
                                                              SEC.
C
   TAU2
           = DAMPING TIME CONSTANT FOR VELOCITY
                                                              SEC.
C
   TD
           = TIME CLOCK WHICH REGULATES DILATATIONAL WAVE
                                                             SEC.
C
           = THE PREVIOUS TD OR TR
   TL
                                                              SEC.
           = TIME CLOCK WHICH REGULATES SHEAR WAVE
C
                                                              SEC.
   TR
C
           = TIME SCALE FACTOR
                                                              SEC.
   TS
C
           = SCALED VALUE OF TIME
   TT
C
   T1
           = DURATION OF MOMENT PULSE AT R≈0
                                                              SEC.
   T3
           = DURATION OF VELOCITY PULSE AT R=0
                                                              SEC.
           = VELOCITY OF DEFLECTION, D(W)/DT - DOWNWARD
                                                              IN/SEC.
C
   VEL
C
  VELS
           = LINEAR VELOCITY SCALE FACTOR
                                                              IN/SEC.
  VELVE
           = SCALED VALUE OF LINEAR VELOCITY
           - TRANSVERSE DISP. OF THE MIDPLANE
                                                              IN.
C
  WS
           - DEFLECTION SCALE FACTOR
           = SCALED VALUE OF DEFLECTION
C WW
С
           = K2*C2
                                                              IN/SEC.
      REAL YB(900), XB(900), YB1(900), YB2(900)
      LOGICAL*1 SYMA/""/,SYMB/","/,SYMC/"."/
      REAL*4 D(6,600)
      INTEGER*2 A(600), B(600), C(600)
      LOGICAL*1 LABEL(40,2)
             MAIN PROGRAM
      DIMENSION R(250), QR(250), VEL(250), W(250), OMEGA(250), QQ(250),
     IVELVE(250), OMOM(250), WW(250), U(250), QQS(250)
      REAL MR(250), MTH(250), NU, MRO, MRS, MTHS, MMR(250), MMTH(250),
     1MMRO, K2
```

```
LOGICAL*1 IDENT(80), FMT(80), NAME(10)
      COMMON T, R, G, EM, IMO, H, DI, RO, RL, MR, MTH, QR, CP, C2, K2, VEL, W, RHO,
     1DEN, NU, OMEGA, MRO, QRO, T1, T3, DSTRN, DALPH, DT, DR, TS, MRS,
     2MTHS, QRS, VELS, OMGS, WS, TT, QQ, MMR, MMTH, VELVE, OMOM, IM, KM, IPR,
     3WW, MMRO, QQO, QQS, Z, U, TAU1, TAU2, ESR, NAME, FMT, IDE NT
      READ 301, IDENT, FMT
      READ 302, NAME, DEN, EM, RO, RL, NU, DR, H
      READ 803, MRO, QRO, T1, T3, TAU1, TAU2
      READ 803, TS, QRS, VELS, WS, MRS, MTHS, OMGS
      READ 804, KM, IPR, KKM, KKM1, KKM2
      READ 805, ESR, DS, IPP
 800 FORMAT(80A1)
 801 FORMAT(80A1/80A1)
 802 FORMAT(10A1,F10.6,E10.2,3F10.2,F10.3,E10.2)
 803 FORMAT(7E10.4)
 804 FORMAT(5110)
 805 FORMAT(2E10.2, I10)
      GEE=386.0
      G=EM/(2.0*(1.0+NU))
      DI=(EM*(H**3))/((12.0)*(1.0-(NU**2)))
      RHO=DEN/GEE
      CP=(EM/(RHO*(1.0-(NU**2))))**0.5
      C2=(G/RHO)**0.5
      K2=((0.76+(0.3*NU))**0.5)
      Z=K2*C2
      DT=DR/CP
C
      DISCRETATION OF PLATE INTO ELEMENTS
      R(1)=R0
      DO 101 I=1,250
      R(I+1)=R(I)+DR
      IF(ABS(R(I+1)-RL)-DR/2.0) 10,10,101
 10
      IM=I
      GO TO 11
      CONTINUE
 101
      IMO=IM+1
 11
      PRINT 900, IDENT
      FORMAT(1HD,80A1,//)
      PRINT 901, NAME, DEN, RHO, EM, G, NU, DI
      PRINT 902, RO, RL, H
      PRINT 903, CP, C2, K2, Z
      PRINT 904, MRO, QRO, T1, T3
      PRINT 905, DT, DR, TS, QRS, VELS, OMGS, MRS, MTHS, WS
      PRINT 906, IM, KM, IPR
      PRINT 9266
 9266 FORMAT(1HS, 2X, 'I', 3X, 'R(I)',/)
      PRINT 907,(I,R(I),I=1,IMO)
      MMRO=MRO/MRS
      QQO=QRO/QRS
      REWIND 11
      REWIND 12
      REWIND 13
      REWIND 20
      REWIND 21
      REWIND 22
```

```
CALL PLATE
      IF(IPP.EQ.1) GO TO 500
C
      PRINT INSTRUCTIONS
      IA=1
      IM2=IPR
 30
      REWIND 11
      REWIND 12
      REWIND 13
      REWIND 20
      REWIND 21
      REWIND 22
      IF(IM+1-IM2) 24,24,25
 24
      IM2=IM+1
      R-T DIAGRAM FOR RADIAL MEMENT
С
 25
      PRINT 908
      DO 104 K=1,KM
      READ(11) TT, MMRO, QQO, (MMR(I), I=1, IMO)
      PRINT FMT, TT, MMRO, QQO, (MMR(I), I=IA, IM2)
 104
      CONTINUE
      R-T DIAGRAM FOR TANGENTIAL MOMENT
      PRINT 4913
      DO 4108 K=1,KM
      READ(13) TT,MMRO,QQO,(MMTH(I),I=1,IMO)
      PRINT FMT, TT, MMRO, QQO, (MMTH(I), I=IA, IM2)
 4108 CONTINUE
      R-T DIAGRAM FOR SHEAR
      PRINT 1909
      DO 105 K=1,KM
      READ(12) TT, MMRO, QQO, (QQ(I), I=1, IMO)
      PRINT FMT, TT, MMRO, QQO, (QQ(I), I=IA, IM2)
 105 CONTINUE
C
      R-T DIAGRAM FOR LINEAR VELOCITY
      PRINT 1910
      DO 106 K=1,KM
      READ(20) TT, MMRO, QQO, (VELVE(I), I=1, IMO)
      PRINT FMT, TT, MMRO, QQO, (VELVE(I), I=IA, IM2)
 106 CONTINUE
      R-T DIAGRAM FOR DEFLECTION
      PRINT 911
      DO 107 K=1,KM
      READ(21) TT, MMRO, QQO, (WW(I), I=1, IMO)
      PRINT FMT, TT, MMRO, QQO, (WW(I), I=IA, IM2)
 107 CONTINUE
      R-T DIAGRAM FOR ANGULAR VELOCITY
      PRINT 1912
      DO 108 K=1,KM
      READ(22) TT, MMRO, QQO, (OMOM(I), I=1, IMO)
      PRINT FMT, TT, MMRO, QQO, (OMOM(I), I=IA, IM2)
 108 CONTINUE
      IF(IM-IM2) 9872,9872,37
 37
      IA=IA+IPR
      IM2=IM2+IPR
      GO TO 30
 9872 PRINT 913
```

```
= ,10A1
   FORMAT(26HOMATERIAL
                                   DEN
            26H WEIGHT DENSITY
                                        = ,F8.3,16H LBS/IN**3
   2
            26H MASS DENSITY
                                   RHO
                                        = ,F8.6,16H LB/IPS**2/IN**4
            26H YOUNGS MODULUS
                                   E = ,E8.2,16H PSI
   3
                                     G
                                       = ,E8.2,16H PSI
            26H MOD. OF RIGIDITY
                                                                     1
                                    NU
                                        = ,F8.2,16H
            26H POISSONS RATIO
            26H FLEXURAL RIGIDITY
                                    D
                                        = ,E8.2,16H LB-IN
902 FORMAT(26HOINNER RADIUS
                                    RO
                                        = ,F8.4,16H IN
            26HOOUTER RADIUS
                                    RL
                                        = ,F8.4,16H IN
                                        = ,F8.4,16H IN
            26HOPLATE THICKNESS
                                     Н
                                                                     1)
903 FORMAT(26HOPLATE VELOCITY
                                    CP
                                        = ,F10.2,16H IN/SEC
            26H SHEAR VELOCITY
                                    C2
                                        = ,F10.2,16H IN/SEC
   1
                                    K2
   2
            26H SHEAR WAVE COR.
                                        = ,F8.6,15H IN**2
   3
            26H WAVE OF SHEAR DISC. Z
                                        = ,F10.2,17H IN/SEC (Z=K2*C2)/)
904 FORMAT(24HOR=0 MOMENT PULSE MRO=
                                          ,F8.0,16H IN-LB
                                          ,F8.0,16H LB
   1
            24H R=O SHEAR PULSE
                                   QRO=
   4
            24H MRO PULSE DURATION T1=
                                          ,E12.5,14H SEC
                                          ,E12.5,14H SEC
            24H QRO PULSE DURATION
                                   T3=
905 FORMAT(26HOTIME INCREMENT
                                    DT
                                       = ,E8.2,16H SEC
   1
            26H CELL LENGTH
                                    DR
                                        = ,F8.5,16H IN
                                        = ,E8.2,16H SEC
   2
            26H TIME SCALE FACTOR
                                    TS
   3
            26H SHEAR SCALE FACTOR QRS
                                        = ,E8.2,16H LB
            26H VEL SCALE FACTOR VELS
                                        = ,E8.2,16H IN/SEC
                                        = ,E8.2,16H RAD/SEC
            26H ANG VEL SCALE FAC OMGS
            26H MOM. SCALE FACTOR MRS
                                        = ,E8.2,16H LB
   7
                                        = ,E8.2,16H LB
            26H MOM. SCALE FACTOR MTHS
            26H DEF SCALE FACTOR
                                    WS
                                        = ,E8.2,16H IN
                                                                     /)
906 FORMAT(26HONO. OF CELLS
                                    IM
                                        = , I4, 15H
            26H TOT. TIME INTERVALS KM
                                        = , 14, 15H
   1
            26H NO. COLS. PRINTED IPR
                                        = ,14,
                                                                  ////)
907 FORMAT(14,F8.4)
908 FORMAT(46H1TIME PULSE
                                                                    1)
                                   RADIAL MOMENT PROPAGATION
1909 FORMAT(46H1TIME PULSE
                                           SHEAR PROPAGATION
                                                                    /)
1910 FORMAT(46H1TIME PULSE
                                 LINEAR VELOCITY PROPAGATION
                                                                    1)
911 FORMAT(46HITIME PULSE
                                      DEFLECTION PROPAGATION
                                                                    1)
                                ANGULAR VELOCITY PROPAGATION
1912 FORMAT(46H1TIME PULSE
                                                                    7)
913 FORMAT(1H1)
4913 FORMAT(46H1TIME PULSE
                                TANGENTIAL MOMENT PROPAGATION
                                                                    1)
500 REWIND 21
     DO 127 K=1,KM
     READ (21) TT,MMRO,QQO,(WW(I),I=1,IMO)
     XB(K)=TT
     YB(K)=WW(1)
     YB1(K)=WW(11)
     YB2(K)=WW(21)
127
    CONTINUE
     CALL INITQ(A,B,C,D,600)
     CALL STSWQ(4662,30)
     READ(5,800) (LABEL(J,1),J=1,40)
     READ(5,800) (LABEL(J,2),J=1,40)
     SET PLOTIT ARGUMENT VALUES.
     N=KM
     LOCX=0
     LOGY=0
```

```
XAXIS=8.0
     YAXIS=5.0
     XMIN=0.0
     NXDEC=5
     XINC=20.0
     YMIN=-10.0
     NYDEC=5
     YINC=4.0
     XSMIN=0.0
     XSMAX=100.0
     YSMIN=-10.0
     YSMAX=10.0
     NDECX=1
     NDECY=1
     HT=0.15
     HTS=0.15
     ICON=0
     CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
    1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
    1 NDECX, NDECY, HTS)
     DO 130 I=1,N
130 YB(I)=YB1(I)
     ICON=0
     CALL SAMEP(YB, N, XB, SYMB, ICON)
     DO 71 I=1,N
71
     YB(I)=YB2(I)
     ICON=0
     CALL SAMEP(YB, N, XB, SYMC, ICON)
     CALL EPLOT
     REWIND 12
     DO 128 K=1,KM
     READ (12) TT, MMRO, QQO, (QQ(I), I=1, IMO)
     XB(K)=TT
     YB(K)=QQ(1)
     YB1(K)=QQ(11)
     YB2(K)=QQ(21)
128 CONTINUE
     CALL INITQ(A,B,C,D,600)
     CALL STSWQ(4662,31)
     READ(5,800) (LABEL(J,1),J=1,40)
     READ(5,800) (LABEL(J,2),J=1,40)
     SET PLOTIT ARGUMENT VALUES.
     N=KM
     LOGX=0
     LOGY=0
     XAXIS=8.0
     YAXIS=5.0
     XMIN=0.0
     NXDEC=5
     XINC=20.0
     YMIN=-10.0
     NYDEC=5
     YINC=4.0
     XSMIN=0.0
```

C

```
XSMAX=100.0
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 72 I=1,N
 72
     YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 73 I=1, N
 73
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 11
      DO 129 K=1,KM
      READ (11) TT,MMRO,QQO,(MMR(I),I=1,IMO)
      XB(K)=TT
      YB(K)=MMR(1)
      YB1(K)=MMR(11)
      YB2(K)=MMR(21)
 129 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,32)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET PLOTIT ARGUMENT VALUES.
      N=KM
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=20.0
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=100.0
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
```

```
1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 74 I=1, N
  74 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 75 I=1, N
75
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 21
      DO 131 K=1,KM
      READ (21) TT,MMRO,QQO,(WW(I), I=1, IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 131
      DO 231 I=1, IMO
      XB(I)=R(I)
      IF(K.EQ.KKM) YB(I)=WW(I)
      IF(K.EQ.KKM1) YB1(I)=WW(I)
      IF(K.EQ.KKM2) YB2(I)=WW(I)
231 CONTINUE
131 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,33)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
С
      SET PLOTIT ARGUMENT VALUES.
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 76 I=1,N
  76 YB(I)=YB1(I)
      ICON-0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
```

```
DO 77 I=1, N
77
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      REWIND 12
      DO 132 K=1,KM
      READ (12) TT, MMRO, QQO, (QQ(I), I=1, IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 132
      DO 232 I=1, IMO
      XB(I)=R(I)
      IF(K.EQ.KKM) YB(I)=QQ(I)
      IF(K.EQ.KKM1) YB1(I)=QQ(I)
      IF(K.EQ.KKM2) YB2(I)=QQ(I)
 232 CONTINUE
 132 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,34)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
      SET PLOTIT ARGUMENT VALUES.
C
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECK, NDECY, HTS)
      DO 78 I=1, N
  78 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 79 I=1, N
 79
      YB(I)=YB2(I)
       ICON=0
       CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
       REWIND 11
```

```
DO 133 K=1,KM
      READ (11) TT, MMRO, QQO, (MMR(I), I=1, IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 133
      DO 233 I=1, IMO
      XB(I)=R(I)
      IF(K.EQ.KKM) YB(I)=MMR(I)
      IF(K.EQ.KKM1) YB1(I)=MMR(I)
      IF(K.EQ.KKM2) YB2(I)=MMR(I)
233 CONTINUE
 133 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,35)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
C
      SET PLOTIT ARGUMENT VALUES.
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN-0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGX, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 81 I=1, N
  81 YB(I)=YB1(I)
      ICON-0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 82 I=1, N
 82
      YB(I)=YB2(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMC, ICON)
      CALL EPLOT
      YB(1)=0.0
      YB1(1)=0.0
      YB2(1)=0.0
      REWIND 21
      DO 136 K=1,KM
      READ (21) TT, MMRO, QQO, (WW(I), I=1, IMO)
      IF(K.NE.KKM.AND.K.NE.KKM1.AND.K.NE.KKM2) GO TO 136
```

```
DO 236 I=1, IM
      XB(I)=R(I)
      IF(K.EQ.KKM) YB(I+1)=(WW(I+1)-WW(I))/DR
      IF(K.EQ.KKM1) YB1(I+1)=(WW(I+1)-WW(I))/DR
      IF(K.EQ.KKM2) YB2(I+1)=(WW(I+1)-WW(I))/DR
 236 CONTINUE
 136 CONTINUE
      CALL INITQ(A,B,C,D,600)
      CALL STSWQ(4662,36)
      READ(5,800) (LABEL(J,1),J=1,40)
      READ(5,800) (LABEL(J,2),J=1,40)
      SET PLOTIT ARGUMENT VALUES.
С
      N=IMO
      LOGX=0
      LOGY=0
      XAXIS=8.0
      YAXIS=5.0
      XMIN=0.0
      NXDEC=5
      XINC=0.2*DS
      YMIN=-10.0
      NYDEC=5
      YINC=4.0
      XSMIN=0.0
      XSMAX=DS
      YSMIN=-10.0
      YSMAX=10.0
      NDECX=1
      NDECY=1
      HT=0.15
      HTS=0.15
      ICON=0
      CALL PLOTIT(YB, N, LOGY, LOGY, XAXIS, YAXIS, XMIN, XINC, NXDEC,
     1 YMIN, YINC, NYDEC, LABEL, XSMIN, XSMAX, YSMIN, YSMAX, XB, SYMA, ICON, HT,
     1 NDECX, NDECY, HTS)
      DO 84 I=1,N
  84 YB(I)=YB1(I)
      ICON=0
      CALL SAMEP(YB, N, XB, SYMB, ICON)
      DO 85 I=1,N
 85
      YB(I)=YB2(I)
      ICON=0
       CALL SAMEP(YB, N, XB, SYMC, ICON)
       CALL EPLOT
       STOP
       END
       SUBROUTINE PLATE
       DIMENSION R(250), QR(250), VEL(250), W(250), OMEGA(250), QQ(250),
      IVELVE(250), OMOM(250), WW(250), AIMR(250), AIMTH(250), U(250), AIQR(250)
      1,QQS(250)
       REAL MR(250), MTH(250), NU, MRO, MRS, MTHS, MMR(250), MMTH(250),
      1MMRO, K2, LIQR(250)
       LOGICAL*1 IDENT(80), FMT(80), NAME(10)
       COMMON T.R.G.EM., IMO, H.DI, RO, RL, MR, MTH, QR, CP, C2, K2, VEL, W, RHO,
```

```
1DEN, NU, OMEGA, MRO, QRO, T1, T3, DSTRN, DALPH, DT, DR, TS, MRS,
      2MTHS, QRS, VELS, OMGS, WS, TT, QQ, MMR, MMTH, VELVE, OMOM, IM, KM, IPR,
      3WW, MMRO, QQO, QQS, Z, U, TAU1, TAU2, ESR, NAME, FMT, IDE NT
      PRINT 921
 921
      FORMAT(1H1)
       INITAL STATE OF PLATE
       DO 119 I=1, IMO
      AIMR(I)=0.0
      AIMTH(I)=0.0
      AIQR(I)=0.0
      LIQR(I)=0.0
      MR(I)=0.0
      MTH(I)=0.0
      QR(I)=0.0
      VEL(I)=0.0
      W(I)=0.0
      OMEGA(I)=0.0
      U(I)=0.0
 119
      CONTINUE
      DTD=DR/CP
      DTR=DR/Z
      T=0.0
      TD=0.0
      TL=0.0
      TLB=0.0
      TR=0.0
      DO 101 K=1,KM
      TT=T/TS
С
      ***INPUT AT R=RO, MOMENT BOUNNDARY CONDITION***
      IF(T-T1) 110,111,111 110 MR(IMO)=MRO
      GO TO 1492 111
                       MR(IMO)=0.0
 1492 CONTINUE
C
      ***INPUT AT R=RO, SHEAR BOUNDARY CONDITION***
      IF(T-T3) 113,114,114
 113
      QR(1)=QRO/(2*3.1415*R(1))
      GO TO 115
 114
      QR(1)=0.00
 115
      CONTINUE
       ***FOUNDATION REACTION***
      DO 116 I=1, IMO
      QQS(I)=-ESR*W(I)
 116 CONTINUE
С
      SCALING FOR PRINTOUT
      IZZ=IMO
      DO 103 I=1, IZZ
      MMR(I)=MR(I)/MRS
      MMTH(I)=MTH(I)/MTHS
      QQ(I)=QR(I)/QRS
      VELVE(I)=VEL(I)/VELS
      OMOM(I)=OMEGA(I)/OMGS
      WW(I)=W(I)/WS
 103 CONTINUE
      WRITE(11)TT, MMRO, QQO, (MMR(I), I=1, IMO)
      WRITE(12)TT, MMRO, QQO, (QQ(I), I=1, IMO)
```

```
WRITE(13)TT,MMRO,QQO,(MMTH(1),I=1,IMO)
      WRITE (20) TT, MMRO, QQO, (75 LVE(1), 1=1, IMO)
      WRITE(21)TT, MMRO, 000, (WW(I), I=1, IMO)
      WRITE(22)TT, MMRO, QQO, (OMOM(I), I=1, IMO)
 12
      IF(ABS(TD-(T+DT))-0.1E-10) 19,19,89
 89
      IF(TD-TR) 15,15,16
      IF(TD+DTD-TR-DTR) 15,15,13
 16
C
      THE CP WAVE
 15
      DO 108 I=2, IMO
      AIMR(I)=AIMR(I)+(MR(I)*(R(I-I)+DR)-MR(I-I)*R(I-I))*DTD
      AIMTH(I)=AIMTH(I)+MTH(I-I)*DR*DTD
      AIQR(I)=AIQR(I)+(QR(I)*(R(I-1)+DR)*DR+QR(I-1)*R(I-1)*DR)*O.5
     1*(TD+DTD-TL)
      LIQR(I-1)=LIQR(I-1)+(QR(I)*(R(I-1)+DR)-QR(I-1)*R(I-1)
     1)*(TD+DTD-TL)
      IF(TAU1-1.0E+40) 930,928,928
 930 DOMG=(-1.0)*OMEGA(I)*(DTD/TAU1)
      OMEGA(I) = OMEGA(I) + (DOMG/2.0)
      IMPULSE AND MOMENTUM ACROSS CP.
C
 928 B=((RHO*H)/24.0)*((H**2))*((2.0*R(I)*DR)+(DR**2))
      DOM=(AIMR(I)-AIMTH(I)-AIQR(I))/B
      OMEGA(I)=OMEGA(I)+DOM
      OMEGA(1)=0.0
      IF(TAU1-1.0E+40) 9113,815,815
      DAMPING, SECOND HALF
 9113 DOMG=(-1.0)*OMEGA(I)*(DTD/TAU1)
      OMEGA(I) = OMEGA(I) + (DOMG/2.0)
      INITIALIZE ANGULAR IMPULSE AFTER USE
 815 \text{ AIMR}(I)=0.0
      AIMTH(I)=0.0
      AIQR(I)=0.0
 108 CONTINUE
      TD=TD+DTD
      TLB=TL
      TL=TD
      DO 107 I=1, IM
      ANGULAR STRAIN
C
      DALPH=((OMEGA(I+1)-OMEGA(I))*DTD)/DR
C
      CONSTITUTIVE EQUATIONS FOR MOMMENT
      IF(I.EQ.1) DMR=DI*(DALPH)
      IF(I.NE.1) DMR=DI*(DALPH+((NU)*U(I-1)))
      DMTH=DI*(U(I)+((NU)*DALPH))
      MR(I)=MR(I)+DMR
      IF(I.EQ.1) MTH(I)=MTH(I)+DMTH
      IF(I.NE.1) MTH(I-1)=MTH(I-1)+DMTH
      U(I)=(OMEGA(I)*DTD)/R(I)
      QR(I+1)=QR(I+1)+(K2**2)*G*H*OMEGA(I+1)*DTD
 107 CONTINUE
      GO TO 12
      DO 104 I=1, IM
 13
      LIQR(I)=LIQR(I)+(QR(I+1)*(R(I)+DR)-QR(I)*R(I))*(TR+DTR-TL)+
     1(0.5*(QQS(I+1)+QQS(I))*(R(I)+DR/2.0)*DR)*DTR
      AIQR(I+1)=AIQR(I+1)+(QR(I+1)*(R(I)+DR)*DR+QR(I)*R(I)*DR)*0.5*(TR+I)
     1DTR-TL)
```

```
IF(TAU2-1.0E+40) 925,931,931
      DAMPING, FIRST HALF
925 DVELG=(-1.0)*/EL(1)*(DTR/TAU2)
     VEL(I)=VEL(I)+(DVELG/2.0)
C
      IMPULSE AND MOMENTUM ACROSS K2*C2
931 A=(R(I)*DR)+((0.5)*DR*DR)
     DVEE=LIQR(I)/(RHO*H*A)
     VEL(I)=VEL(I)+DVEE
     VEL(IM+1)=0.0
      IF(TAU2-1.0E+40) 927,818,818
      DAMPING, SECOND HALF
927 DVELG=(-1.0)*VEL(I)*(DTR/TAU2)
     VEL(I)=VEL(I)+(DVELG/2.0)
818 LIOR(I)=0.0
 104 CONTINUE
      TR=TR+DTR
      TLB=TL
      TL=TR
      DO 102 I=1, IM
      DSTRN=((VEL(I+1)-VEL(I))*DTR)/DR
      DOR=(K2**2)*G*H*(DSTRN)
      QR(I+1)=QR(I+1)+DQR
      W(I)=W(I)+(YEL(I)*DTR)
 102 CONTINUE
      GO TO 12
 19
      CONTINUE
      T=T+DT
 101 CONTINUE
      ENDFILE 11
      ENDFILE 12
      ENDFILE 13
      ENDFILE 20
      ENDFILE 21
      ENDFILE 22
 500 RETURN
      END
//DATA.FT11F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.FT12F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.FT13F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.FT20F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.FT21F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.FT22F001 DD UNIT=SYSDA, SPACE=(TRK, (5,5), RLSE)
//DATA.INPUT DD *
CONDITION 1 FOR PLATE ON ELASTIC FOUNDATION
(1HS,F6.2,1X,F5.1,1X,F5.1,1X,22F5.2)
              0.0868
                      30.00E+5
                                      0.20
                                               15.00
                                                           0.20
                                                                      0.20
 CONCRETE
4.00
  0.000E+0 -100.0E+0 4.0000E+0 4.0000E+0 3.0000E-5 3.0000E-5
 1.0000E-5 10.000E+0 4.0000E-1 5.0000E-6
                                            5.00E+0 1.0000E+0 5.0000E+0
                  22
                            100
                                      100
                                                 50
       100
   1.20E+5 15.00E+0
TIME IN 1.0*E-5 SEC
PLATE DEFLECTION IN 5*E-6 INCHES
TIME IN 1.0*E-5 SEC
```

```
PLATE SHEAR IN 10.*LB.
TIME IN 1.0*E-5 SEC
PLATE MOMENT IN 5.*LB-IN.
DISTANCE X IN INCHES
PLATE DEFLECTION IN 5*E-6 INCHES
DISTANCE X IN INCHES
PLATE SHEAR IN 10.*LB
DISTANCE X IN INCHES
PLATE MOMENT IN 5.*LB-IN.
DISTANCE X IN INCHES
PLATE SLOPE IN 5*E-6
//DATA.FT30F001 DD DSN=MEN.P11850.A3B.PLOT,
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
11
11
               SPACE=(TRK,(1,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT31F001 DD DSN=MEN.P11850.A3B.PLOT1,
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
11
               SPACE=(TRK,(1,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT32F001 DD DSN=MEN.P11850.A3B.PLOT2,
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
11
               SPACE=(TRK,(1,2),RLSE),
               DCB=(RECFM=FB, LRECL=90, BLKSIZE=12960)
11
//DATA.FT33F001 DD DSN=MEN.P11850.A3B.PLOT3,
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
//
//
               SPACE=(TRK,(1,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT34F001 DD DSN=MEN.P11850.A3B.PLOT4,
11
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
11
               SPACE=(TRK,(1,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT35F001 DD DSN=MEN.P11850.A3B.PLOT5,
               VOL=REF=MEN.P11850.A3B.LIB, DISP=(NEW, KEEP),
//
//
               SPACE=(TRK,(1,2),RLSE),
11
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
//DATA.FT36F001 DD DSN=MEN.P11850.A3B.PLOT6,
               VOL=REF=MEN.P11850.A3B.LIB,DISP=(NEW,KEEP),
//
11
               SPACE=(TRK,(1,2),RLSE),
               DCB=(RECFM=FB, LRECL=80, BLKSIZE=12960)
11
```

